UVIS - ISPITNI ROK 2.9.2021.

1. HIPOTEZE:

$H_1 = \{$ odabrani student je odličan $\}$ $\Rightarrow$ $P(H_1) = \dfrac{a}{a+b+c}$ | $P(A|H_1) = 1$

$H_2 = \{$ odabrani student je vrlo dobar $\}$ $\qquad P(H_2) = \dfrac{b}{a+b+c}$ | $P(A|H_2) = 1$

$H_3 = \{$ odabrani student je dobar $\}$ $\qquad P(H_3) = \dfrac{c}{a+b+c}$ | $P(A|H_3) = \dfrac{1}{3}$

$A = \{$ student je dobio odličnu ili vrlo dobru ocjenu $\}$

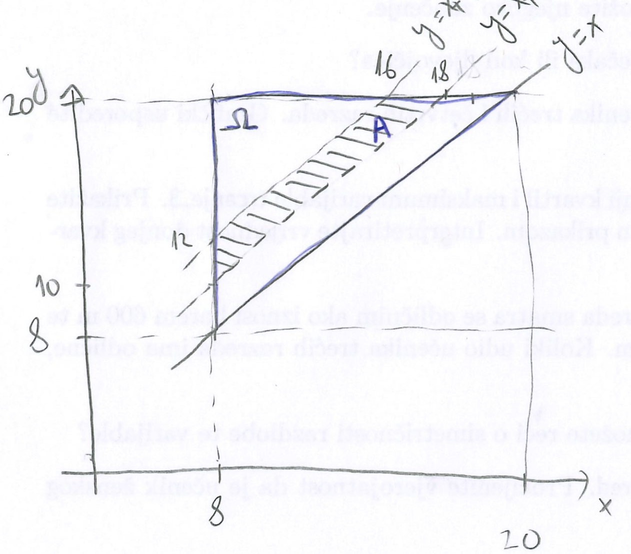
$$P(H_2|A) = ?$$

$$P(A) \overset{FPV}{=} \sum_{i=1}^{3} P(H_i)\cdot P(A|H_i) = \frac{a+b+c/3}{a+b+c} = \frac{3a+3b+c}{3(a+b+c)}$$

Bayesova formula:

$$\Rightarrow P(H_2|A) = \frac{P(A|H_2)\cdot P(H_2)}{P(A)} = \frac{1\cdot\frac{b}{a+b+c}}{\frac{3a+3b+c}{3(a+b+c)}} = \frac{3b}{3a+3b+c}$$

2.



$$\lambda(\Omega) = \frac{(20-8)(20-8)}{2} = 72$$

$$\lambda(A) = \frac{(18-8)(18-8)}{2} - \frac{(16-8)(16-8)}{2} = 50 - 32 = 18$$

$$\Rightarrow P(A) = \frac{\lambda(A)}{\lambda(\Omega)} = \frac{18}{72} = \frac{1}{4}$$

3.

$$f_X(x) = \begin{cases} \dfrac{\alpha}{x^{1+\alpha}} e^{-x^{-\alpha}}, & x>0 \\ 0, & x\le 0 \end{cases}, \quad \alpha>0, \quad Y = \frac{1}{X}, \quad f_Y(y) = ?, \quad E[Y] = ?$$

$$g(x) = \frac{1}{x} \Rightarrow Y = g(X), \ g \text{ je bijekcija na } \langle 0,\infty\rangle, \ g^{-1}(x) = \frac{1}{x}, \ [g^{-1}(x)]' = -\frac{1}{x^2}$$

$$\Rightarrow f_Y(y) = \begin{cases} f_X(g^{-1}(y))\cdot|[g^{-1}(y)]'|, & y\in \mathcal{R}(g) \\ 0, & y\notin \mathcal{R}(g) \end{cases} = \begin{cases} \alpha y^{1+\alpha} e^{-y^{\alpha}} \dfrac{1}{y^2}, & y>0 \\ 0, & y\le 0 \end{cases} = \begin{cases} \alpha y^{\alpha-1} \dots \end{cases}$$

$$\Rightarrow f_Y(y) = \begin{cases} \alpha y^{\alpha-1} e^{-y^\alpha}, & y>0 \\ 0, & y \le 0 \end{cases}$$

$$E[Y] = \int_0^\infty y \alpha y^{\alpha-1} e^{-y^\alpha} dy = \left| \begin{matrix} y^\alpha = t \\ \alpha y^{\alpha-1} dy = dt \end{matrix} \right| = \int_0^\infty t^{1/\alpha} e^{-t} dt = \int_0^\infty t^{(1+1/\alpha)-1} e^{-t} dt = \Gamma(1+1/\alpha)$$

4.

X - s.v. kojom modeliramo broj petica, $R(X) = \{0,1,2\}$

Y - s.v. kojom modeliramo zbroj brojeva koji padnu, $R(Y) = \{2,3,4,\dots,12\}$

- tablica distribucije:

X\Y	2	3	4	5	6	7	8	9	10	11	12	
0	1/36	2/36	3/36	4/36	3/36	4/36	3/36	2/36	2/36	0	1/36	25/36
1	0	0	0	0	2/36	2/36	2/36	2/36	0	2/36	0	10/36
2	0	0	0	0	0	0	0	0	1/36	0	0	1/36
	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	1

- marginalne distribucije:

$$X = \begin{pmatrix} 0 & 1 & 2 \\ \frac{25}{36} & \frac{10}{36} & \frac{1}{36} \end{pmatrix}, \quad Y = \begin{pmatrix} 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ \frac{1}{36} & \frac{2}{36} & \frac{3}{36} & \frac{4}{36} & \frac{5}{36} & \frac{6}{36} & \frac{5}{36} & \frac{4}{36} & \frac{3}{36} & \frac{2}{36} & \frac{1}{36} \end{pmatrix}$$

- $P(XY=20) = P(X=2, Y=10) = 1/36$

5.

$$X = \begin{pmatrix} 1 & 2 & 3 & 4 & \dots & n & \dots \\ p & p^2 & p^3 & p^4 & \dots & p^n & \dots \end{pmatrix} \Rightarrow X = \begin{pmatrix} 1 & 2 & \dots & n \\ \frac{1}{2} & \frac{1}{4} & \dots & \left(\frac{1}{2}\right)^n & \dots \end{pmatrix}$$

a) $p = ?$

$$\sum_{k=1}^\infty p^k = 1 \Rightarrow \frac{p}{1-p} = 1 \Rightarrow \boxed{p = 1/2}$$

b) $E[X] = \sum_{k=1}^\infty k \left(\frac{1}{2}\right)^k$

funkcija izvodnica: $g_X(z) = \sum_{k=1}^\infty z^k p_k = \sum_{k=1}^\infty z^k \left(\frac{1}{2}\right)^k$

$$= \frac{z/2}{1-z/2} = \frac{z}{2-z}$$

$$\Rightarrow g_X'(z) = \frac{2}{(2-z)^2} \Rightarrow g_X'(1) = 2$$

$$\Rightarrow E[X] = g_X'(1) = 2$$