UVIS - ISPITNI ROK 28.6.2021.

1. $A = \{$ izvučena su tri asa $\}$, $B = \{$ izvučena su tri kralja $\}$, $P(A\cup B) = ?$

$k(\Omega) = \binom{52}{8}$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$= \frac{\binom{4}{3}\binom{48}{5}}{\binom{52}{8}} + \frac{\binom{4}{3}\binom{48}{5}}{\binom{52}{8}} - \frac{\binom{4}{3}\binom{4}{3}\binom{44}{2}}{\binom{52}{8}}$$

2. $(\Omega, \mathcal{F}, P)$, $A, B \in \mathcal{F}$

Imamo:

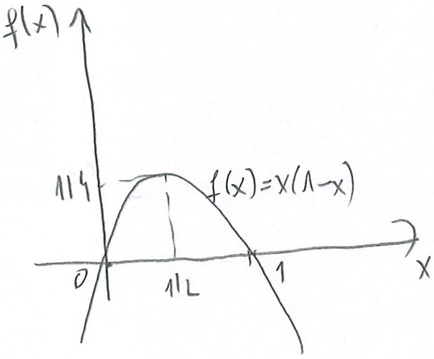
$$P(A)(1+P(B)) + P(B)(1+P(A)) - (P(A)+P(B))^2 = P(A) + P(B) + \cancel{2P(A)P(B)} - P(A)^2 - \cancel{2P(A)P(B)} - P(B)^2$$

$$= P(A)(1-P(A)) + P(B)(1-P(B))$$

$$\overset{*}{\leq} \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \;//$$

Kako su $P(A), P(B) \in [0,1]$

i $f(x) = x(1-x) \leq \frac{1}{2}\left(1-\frac{1}{2}\right) = \frac{1}{4}$



3. Izračunajmo najprije funkciju distribucije s.v. X s funkcijom gustoće

$f(x) = \frac{1}{2}e^{-|x|}$, $x \in \mathbb{R}$:

$$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt = \int_{-\infty}^{x} \frac{1}{2}e^{-|t|}\,dt$$

- $x \in \langle -\infty, 0\rangle$: $F_X(x) = \int_{-\infty}^{x} \frac{1}{2}e^{t}\,dt = \frac{1}{2}e^{t}\Big|_{-\infty}^{x} = \frac{1}{2}e^{x}$
- $x \in [0, +\infty\rangle$: $F_X(x) = \int_{-\infty}^{x} \frac{1}{2}e^{-|t|}\,dt = \int_{-\infty}^{0} \frac{1}{2}e^{t}\,dt + \int_{0}^{x} \frac{1}{2}e^{-t}\,dt = \frac{1}{2}e^{t}\Big|_{-\infty}^{0} + \frac{1}{2}(-e^{-t})\Big|_{0}^{x}$

$$= \frac{1}{2} - \frac{1}{2}e^{-x} + \frac{1}{2} = 1 - \frac{1}{2}e^{-x} \;//$$

$\Rightarrow F_X(x) = \begin{cases} \frac{1}{2}e^{x}, & x<0 \\ 1-\frac{1}{2}e^{-x}, & x\geq 0 \end{cases}$ $\Rightarrow F_Y(y) = P(Y \leq y) = P(2X+1 \leq y) = F_X\left(\frac{y-1}{2}\right)$

$$= \begin{cases} \frac{1}{2}e^{\frac{y-1}{2}}, & y<1 \\ 1-\frac{1}{2}e^{-\frac{y-1}{2}}, & y\geq 1 \end{cases}$$

• $E[Y] = E[2X+1] \overset{\text{lin.č.}}{=} 2E[X]+1$

$$= 2\int_{\mathbb{R}} x f(x)\,dx + 1 = \cancel{2}\int_{\mathbb{R}} x \frac{1}{\cancel{2}} e^{-|x|}\,dx + 1$$

$$= \int_{-\infty}^{0} x e^{x}\,dx + \int_{0}^{\infty} x e^{-x}\,dx + 1 = \underbrace{x e^{x}\Big|_{-\infty}^{0}}_{=0} - \int_{-\infty}^{0} e^{x}\,dx - \underbrace{x e^{-x}\Big|_{0}^{\infty}}_{=0}$$

$$+ \int_{0}^{\infty} e^{-x}\,dx + 1$$

$$= -e^{x}\Big|_{-\infty}^{0} - e^{-x}\Big|_{0}^{\infty} + 1 = -1+1+1 = 1$$

(ili kraće: $g(x) = \frac{1}{2}xe^{-|x|}$ je neparna funkcija, a integral neparne funkcije na sim. intervalu je 0 $\Rightarrow \int_{\mathbb{R}} \frac{1}{2}xe^{-|x|}dx = 0 \Rightarrow E[X]=0 \Rightarrow E[Y] = 2E[X]+1$
$= 2\cdot 0+1 = 1$)

4. Imamo: $E[Y]=-1$, $E[X]=2$, $\mathrm{Var}X=9$, $\mathrm{Var}Y=4$, $\rho_{XY}=0{,}5$, $Z=2XY+1$

$E[Z]=?$

$\Rightarrow E[Z] = 2E[XY]+1$

Kako je $\rho_{X,Y} = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}X\,\mathrm{Var}Y}} = \frac{E[XY]-E[X]\cdot E[Y]}{\sqrt{\mathrm{Var}X\,\mathrm{Var}Y}} = \frac{E[XY]+2}{6} \Rightarrow E[XY] =$
$= 6\rho_{XY}-2$
$= 6\cdot 0{,}5-2$
$= 1$

$\Rightarrow E[Z] = 2\cdot 1+1 = 3$

5. vidi prethodni rok!