UVIS – 1. kolokvij 16./17. RJEŠENJA (B)

(2.) $P(B)>0,\ P(A|B^c)-P(A|B)=0 \iff \frac{P(A\cap B^c)}{P(B^c)}-\frac{P(A\cap B)}{P(B)}=0$

$\iff P(B)\,P(A\cap B^c)-(1-P(B))\,P(A\cap B)=0$

$\iff P(A\cap B)=P(B)\left(P(A\cap B^c)+P(A\cap B)\right)=P(B)\cdot P(A\cap\Omega)=P(B)P(A)$

$\Rightarrow$ A i B nezavisni.

(3.) • $A=\{$ niz izvučenu miti jedna hrvatska kartu$\}$, $k(\Omega)=\binom{52}{6}$

$$P(A)=\frac{\binom{39}{6}}{\binom{52}{6}}$$

• $B=\{$ izvučene je 2 crne ili 2 pika karte$\}$

$B_1=\{$ izvučene je 2 crne karte$\}$, $B_2=\{$ izvučene je 2 pika karte$\}$

$B_1\cup B_2=B$, $B_2\subset B_1 \Rightarrow B_1\cup B_2=B_1$

$$\Rightarrow P(B)=P(B_1\cup B_2)=P(B_1)=\frac{\binom{26}{2}\binom{26}{4}}{\binom{52}{6}}\ //$$

(4.) $A=\{$ prva komponenta je ispravna$\}$, $B=\{$ druga komponenta je ispravna$\}$

$P(A)=0.06$, $P(B)=0.07$, A & B nezavisni.

$P(\{$ sustav ispravno počinje raditi$\})=P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\overset{\text{nezavisnost}}{=} P(A)+P(B)-P(A)\cdot P(B)=0.1258\ //$

(5.) $H_i = \{$ početni broj zelenih kuglica je $i\}$, $i \in \{0,1,\dots,n\}$

$P(H_0) = P(H_1) = \dots = P(H_n) = \frac{1}{n+1}$

$A = \{$ prilikom izvlačenja je izvučena zelena kuglica $\}$ $\Big\}$ $P(A|H_i) = \frac{i+1}{n+1}$

- $P(A) \overset{FPV}{=} \sum_{i=0}^{n} P(A|H_i)P(H_i) = \left(\frac{1}{n+1}\right)^2 \sum_{i=0}^{n}(i+1) = \boxed{\frac{n+2}{2(n+1)}}$

- $n=4$:

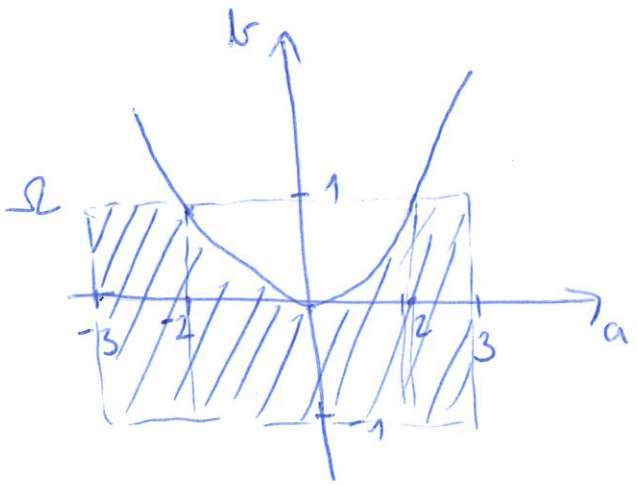
$$P(H_i|A) = \frac{P(A|H_i)P(H_i)}{P(A)} = \frac{\frac{i+1}{n+1}\cdot\frac{1}{n+1}}{\frac{n+2}{2(n+1)}} = \frac{2(i+1)}{(n+1)(n+2)} = \frac{2(i+1)}{5\cdot 6} = \frac{i+1}{15}$$

$\Rightarrow P(H_0|A) < P(H_1|A) < \dots < P(H_4|A) = \frac{5}{15} = \frac{1}{3}$ //

najvjerojatniji početni sastav zelenih kuglica je 4.

(6.) $x^2 + ax + b = 0$: rješenja realna $\Leftrightarrow D = a^2 - 4b \geq 0 \Leftrightarrow b \leq \frac{a^2}{4}$

$a \in [-3,3]$, $b \in [-1,1]$, $\Omega = [-3,3] \times [-1,1]$




$A = \{$ rješenja su realna $\}$

$$P(A) = \frac{\lambda(A)}{\lambda(\Omega)} = \frac{\lambda(\Omega) - \lambda(A^c)}{\lambda(\Omega)}$$

$$= 1 - \frac{\int_{-2}^{2}\left(1 - \frac{a^2}{4}\right)da}{6\cdot 2}$$

$$= 1 - \frac{4 - \frac{a^3}{12}\Big|_{-2}^{2}}{6\cdot 2}$$

$$= 1 - \frac{4 - 2\cdot\frac{8}{12}}{12}$$

$$= 1 - \frac{8}{36} = \frac{28}{36} = \boxed{\frac{7}{9}}$$ //