UVIS: 2017./2018. – 1. kolokvij (A)

2. $P(A\cap B) = P(A)+P(B)-P(A\cup B)$

$\Rightarrow P(A\cap B) - P(B) = P(A) - P(A\cup B)$ $\quad \left(\begin{array}{c} P(A\cup B)\le 1 \\ \Rightarrow -P(A\cup B)\ge -1 \end{array}\right)$

$\ge P(A) - 1.$ //

3. $X = \begin{pmatrix} -1 & 0 & 1 \\ 2p^2 & p & 4p^2 \end{pmatrix}$

a) $2p^2+p+4p^2=1 \Rightarrow 6p^2+p-1=0 \Rightarrow p_{1,2} = \frac{-1\pm\sqrt{1+4\cdot 6}}{2\cdot 6} = \frac{-1\pm 5}{12}$

$\Rightarrow \boxed{p=\frac{1}{3}}$ $\qquad \Rightarrow (p_1 = \frac{1}{3}),\ p_2 = \cancel{-\frac{1}{2}}$

b) $X = \begin{pmatrix} -1 & 0 & 1 \\ \frac{2}{9} & \frac{1}{3} & \frac{4}{9} \end{pmatrix}$

$Y = |X| - 1$, $g(x) = |x| - 1$ $\Rightarrow$ $g(-1)=0,\ g(0)=-1,\ g(1)=0$ $\Rightarrow Y = \begin{pmatrix} 0 & -1 \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

4. a) $A = \{$izvukli smo 3 kuglice s neparnim brojem$\}$

$P(A) = \frac{13\cdot 12\cdot 11}{25\cdot 24\cdot 23} = \frac{13\cdot 11}{50\cdot 23} = \frac{143}{1150}$

b) $B = \{$izvukli smo 3 kuglice s uzastopnim brojem u rastućem poretku$\}$

$P(B) = \frac{23\cdot 1\cdot 1}{25\cdot 24\cdot 23} = \frac{1}{600}$

5. $A = \{$niti jedna kuglica nije plave boje$\}$

$H_1 = \{$obje kuglice potječu iz prve kutije$\}$

$H_2 = \{$obje kuglice potječu iz druge kutije$\}$

$H_3 = \{$obje kuglice potječu iz treće kutije$\}$

$H_4 = \{$jedna kuglica iz prve, jedna iz druge$\}$

$H_5 = \{$jedna kuglica iz prve, jedna iz treće$\}$

$H_6 = \{$jedna iz druge, jedna iz treće$\}$

$P(H_1) = \frac{\binom{5}{2}}{\binom{14}{2}}$, $P(H_2) = \frac{\binom{5}{2}}{\binom{14}{2}}$, $P(H_3) = \frac{\binom{4}{2}}{\binom{14}{2}}$

$P(H_4) = \frac{\binom{5}{1}\binom{5}{1}}{\binom{14}{2}}$, $P(H_5) = \frac{\binom{5}{1}\binom{4}{1}}{\binom{14}{2}}$, $P(H_6) = \frac{\binom{5}{1}\binom{4}{1}}{\binom{14}{2}}$

$P(A|H_1) = \frac{\binom{2}{2}}{\binom{5}{2}}$, $P(A|H_2) = \frac{\binom{3}{2}}{\binom{5}{2}}$, $P(A|H_3) = \frac{\binom{2}{2}}{\binom{4}{2}}$

$P(A|H_4) = \frac{\binom{2}{1}\binom{3}{1}}{\binom{5}{1}\binom{5}{1}}$, $P(A|H_5) = \frac{\binom{2}{1}\binom{2}{1}}{\binom{5}{1}\binom{4}{1}}$, $P(A|H_6) = \frac{\binom{3}{1}\binom{2}{1}}{\binom{5}{1}\binom{4}{1}}$

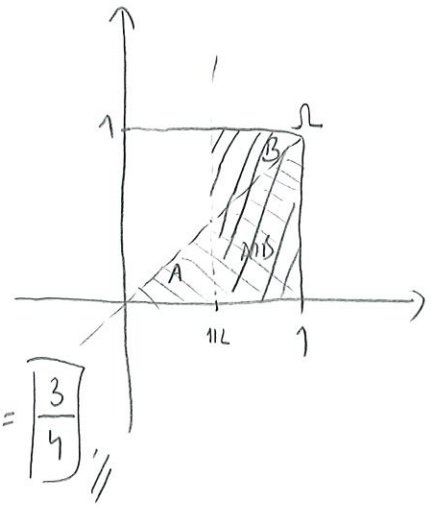
$\Rightarrow P(A) = \sum_{i=1}^{6} P(A|H_i)\,P(H_i) = \frac{1}{\binom{14}{2}}\left[1+3+1+\cancel{6}+4+\cancel{6}\right] = \frac{11}{91} \approx 0{,}1307$

(6.) $x, y \in [0,1]$, $\Omega = [0,1]^2 \Rightarrow \lambda(\Omega) = 1$

$A = \{(x,y) \in \Omega : x > y\}$, $B = \{(x,y) \in \Omega : x > 1/2\}$

$P(B|A) = ?$

$$\Rightarrow P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{\lambda(A \cap B)}{\lambda(\Omega)}}{\frac{\lambda(A)}{\lambda(\Omega)}} = \frac{\lambda(A \cap B)}{\lambda(A)} = \frac{\frac{1}{2} \cdot 1 - \frac{\frac{1}{2} \cdot \frac{1}{2}}{2}}{1/2}$$



$$= \frac{\frac{1}{2} - \frac{1}{8}}{\frac{1}{2}} = \frac{\frac{3}{8}}{\frac{1}{2}} = \boxed{\frac{3}{4}} \; //$$

(B)

(2.) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (*)

$\Rightarrow$ Traženu nejednakost ekvivalentno je sq:

$$2P(A \cup B) \geq P(A) + P(B)$$

$$\overset{(*)}{\Leftrightarrow} 2P(A) + 2P(B) - 2P(A \cap B) \geq P(A) + P(B)$$

$$\Leftrightarrow P(A) + P(B) - 2P(A \cap B) \geq 0$$

$$\overset{(*)}{\Leftrightarrow} P(A \cup B) \geq P(A \cap B) \quad \checkmark\checkmark \quad (\text{jer je } A \cap B \subseteq A \cup B)$$

time smo dokazali traženu tvrdnju!

(3.) $X = \begin{pmatrix} -1 & 0 & 1 \\ p & 5p^2 & 7p^2 \end{pmatrix}$

a) $p + 5p^2 + 7p^2 = 1 \Rightarrow 12p^2 + p - 1 = 0 \Rightarrow p_{1,2} = \frac{-1 \pm \sqrt{1 + 12 \cdot 4}}{2 \cdot 12} = \frac{-1 \pm 7}{24} \Rightarrow \boxed{p_1 = \frac{1}{4}}$ ~~$p_2 = -\frac{1}{3}$~~

$\Rightarrow p = \frac{1}{4}$ //

b) $X = \begin{pmatrix} -1 & 0 & 1 \\ \frac{1}{4} & \frac{5}{16} & \frac{7}{16} \end{pmatrix}$

$Y = |X| - 1$, $g(x) = x - 1$ $\Rightarrow$ $g(-1) = 0$, $g(0) = -1$, $g(1) = 0$

$$Y = \begin{pmatrix} 0 & -1 \\ \frac{11}{16} & \frac{5}{16} \end{pmatrix}$$

(4.) a) $A = \{$izvukli smo 4 kuglice s parnim brojevima$\}$

$$P(A) = \frac{12 \cdot 11 \cdot 10 \cdot 9}{25 \cdot 24 \cdot 23 \cdot 22} = \frac{90}{100 \cdot 23} = \frac{9}{230} \; //$$

b) $B = \{$izvukli smo 4 kuglice s uzastopnim brojevima u rastućem poretku$\}$

$$P(B) = \frac{22 \cdot 1 \cdot 1 \cdot 1}{25 \cdot 24 \cdot 23 \cdot 22} = \frac{1}{13800} \; //$$

(5.) $A = \{$ barem ena kroglica je bizele barve $\} \Rightarrow A^c = \{$ niti ena kroglica ni bizele $\}$

$H_1 = \{$ obe kroglici potegnem iz prve kutije $\}$

$H_2 = \{$ obe kroglici potegnem iz druge kutije $\}$

$H_3 = \{$ obe kroglici potegnem iz tretje kutije $\}$

$H_4 = \{$ ena kroglica iz prve, ena iz druge $\}$

$H_5 = \{$ ena kroglica iz prve, ena iz tretje $\}$

$H_6 = \{$ ena kroglica iz druge, ena iz tretje $\}$

$$P(H_1) = \frac{\binom{4}{2}}{\binom{14}{2}}, \quad P(H_2) = \frac{\binom{5}{2}}{\binom{14}{2}}, \quad P(H_3) = \frac{\binom{5}{2}}{\binom{14}{2}}$$

$$P(H_4) = \frac{\binom{4}{1}\binom{5}{1}}{\binom{14}{2}}, \quad P(H_5) = \frac{\binom{4}{1}\binom{5}{1}}{\binom{14}{2}}, \quad P(H_6) = \frac{\binom{5}{1}\binom{5}{1}}{\binom{14}{2}}$$

$$P(A^c|H_1) = \frac{\binom{2}{2}}{\binom{4}{2}}, \quad P(A^c|H_2) = \frac{\binom{2}{2}}{\binom{5}{2}}, \quad P(A^c|H_3) = \frac{\binom{3}{2}}{\binom{5}{2}}$$

$$P(A^c|H_4) = \frac{\binom{2}{1}\binom{2}{1}}{\binom{4}{1}\binom{5}{1}}, \quad P(A^c|H_5) = \frac{\binom{2}{1}\binom{3}{1}}{\binom{4}{1}\binom{5}{1}}, \quad P(A^c|H_6) = \frac{\binom{2}{1}\binom{3}{1}}{\binom{5}{1}\binom{5}{1}}$$

$$\Rightarrow P(A) = 1 - P(A^c) = 1 - \sum_{i=1}^{6} P(H_i) P(A^c|H_i)$$

$$= 1 - \frac{1}{\binom{14}{2}}[1+1+3+4+6+6] = 1 - \frac{21}{91} = 1 - \frac{3}{13} = \boxed{\frac{10}{13}} \; //$$

$\approx (0.769230769)$

(6.) $x, y \in [0,1]$, $\Omega = [0,1]^2$, $\lambda(\Omega) = 1$

$A = \{(x,y) \in \Omega : x < y\}$, $B = \{(x,y) \in \Omega : y < 1/2\}$

$P(B|A) = ?$

$$\Rightarrow P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\left(\frac{1}{2} \cdot \frac{1}{2}\right)/2}{(1 \cdot 1)/2} = \frac{1}{4} \; //$$