2.

a) $X \sim B(72, \frac{1}{6})$ 2 $\Rightarrow X = \begin{pmatrix} 0 & 1 & \cdots & 72 \\ \left(\frac{5}{6}\right)^{72} & 72 \cdot \left(\frac{1}{6}\right)^{71} \frac{5}{6} & \cdots & \left(\frac{1}{6}\right)^{72} \end{pmatrix}$ 2

b) $P(|X-12| \geq 2\sqrt{10}) = P(X \in \langle -\infty, 12-2\sqrt{10}] \cup [12+2\sqrt{10}, +\infty)) =$ 2

$= P(X \leq 12-2\sqrt{10}) + P(X \geq 12+2\sqrt{10}) = \sum_{k=0}^{5} P(X=k) + \sum_{k=19}^{72} P(X=k)$ 2

$= 1 - \sum_{k=6}^{18} P(X=k) = \boxed{1 - \sum_{k=6}^{18} \binom{72}{k} \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{72-k}}$

Čebiševljeva nejednakost:

$P(|X-12| \geq 2\sqrt{10}) \leq \frac{1}{2^2} = \frac{1}{4}$. 2

3.

$p_n = 0.1n + 0.3 \Rightarrow p_1 = 0.4, \; p_2 = 0.5, \; p_3 = 0.6$

$\mathcal{R}(X) = \{0, 1, 2, 3\}$

$P(X=0) = (1-p_1)(1-p_2)(1-p_3) = 0.12 = \frac{3}{25}$

$P(X=1) = p_1(1-p_2)(1-p_3) + (1-p_1)p_2(1-p_3) + (1-p_1)(1-p_2)p_3 = 0.38 = \frac{19}{50}$

$P(X=2) = p_1 p_2 (1-p_3) + p_1(1-p_2)p_3 + (1-p_1)p_2 p_3 = 0.38 = \frac{19}{50}$

$P(X=3) = p_1 p_2 p_3 = 0.12 = \frac{3}{25}$ $\Rightarrow X = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.12 & 0.38 & 0.38 & 0.12 \end{pmatrix}$ 4

$EX = 0 \cdot 0.12 + 1 \cdot 0.38 + 2 \cdot 0.38 + 3 \cdot 0.12 = 1.5$ 3

$= \frac{3}{2}$

4.

$X \sim \mathcal{U}(a, b)$

$EX = 4, \; \sigma_X = 2\sqrt{3}$

$\left.\begin{aligned} EX &= \frac{a+b}{2} = 4 \\ \sigma_X^2 &= \frac{(b-a)^2}{12} = 12 \end{aligned}\right\} \quad \begin{aligned} a+b &= 8 \\ b-a &= 12 \end{aligned}$ 4 $\Rightarrow \boxed{b = 10, \; a = -2}$

$EX^3 = \int_{-2}^{10} x^3 f(x)\, dx$

$= \int_{-2}^{10} \frac{x^3}{12} dx = \frac{x^4}{48} \Big|_{-2}^{10}$ 4

$= \frac{1}{48}(10^4 - 2^4) = 208$

(5.) $f_X(x) = \frac{1}{\pi(1+x^2)}, \ x \in \mathbb{R} \Rightarrow F(x) = P(X \le x) = \int_{-\infty}^{x} \frac{1}{\pi(1+t^2)}\,dt = \frac{1}{\pi}\operatorname{arctg} x + \frac{1}{2}$

$Y = 1/X \Rightarrow g^{-1}(x) = \frac{1}{x} \Rightarrow [g^{-1}(x)]' = -\frac{1}{x^2}$

$$f_Y(y) = \begin{cases} f_X(g^{-1}(y)) \cdot \left|[g^{-1}(y)]'\right|, & y \in \mathcal{R}(g) \\ 0, & \text{inače} \end{cases}$$

$$= \begin{cases} f_X\left(\frac{1}{y}\right) \cdot \frac{1}{y^2}, & y \in \mathbb{R} \\ 0, & \text{inače} \end{cases} = \frac{1}{\pi\left(1+\frac{1}{y^2}\right)} \cdot \frac{1}{y^2} = \boxed{\frac{1}{\pi(1+y^2)}}, \ y \in \mathbb{R}$$ 5

$\Rightarrow$ Y ima istu dist. kao X

$$\Rightarrow \boxed{F_Y(y) = \frac{1}{\pi}\operatorname{arctg} y + \frac{1}{2}}$$ 5

(6.) X – broj neparnih brojeva, Y – broj pojavljivanja zelene boje

$\mathcal{R}(X) = \mathcal{R}(Y) = \{0, 1, 2\}$

1. kockica	2. kockica
nep.: zeleno	nep.: žuto
parno: žuto	parno: zeleno

X\Y	0	1	2	
0	0	$\frac{1}{4}$	0	$\frac{1}{4}$
1	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$
2	0	$\frac{1}{4}$	0	$\frac{1}{4}$
	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	

4

2

$$X \overset{d}{=} Y = \begin{pmatrix} 0 & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix}$$

$$\rho_{X,Y} = \frac{Cov(X,Y)}{\sqrt{VarX \cdot VarY}} = \frac{E[XY] - E[X]\cdot E[Y]}{\sqrt{VarX \cdot VarY}} = \frac{1 - 1\cdot 1}{\frac{1}{2}} = 0$$ 2

$\boxed{EX = EY = 1}$ $\quad VarX = EX^2 - (EX)^2 = \frac{3}{2} - 1^2 = \frac{1}{2}$

$\mathcal{R}(XY) = \{0, 2\}$

$P(XY = 0) = P(X=0, Y=1) + P(X=1, Y=0) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$P(XY = 2) = P(X=1, Y=2) + P(X=2, Y=1) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$E[XY] = 1$

X i Y su nekorelirane! 2