UNIT 5 Equations and formulae

Reading

Section 1 Equations

If we wish to solve an equation, we must find the value of the letter (usually x) which satisfies the equation. When the equation is solved, the answer must be checked, by substituting it for x in the original equation.

EXAMPLE

$$x + 9 = 23$$

Here we subtract 9 from each side;

$$x + 9 - 9 = 23 - 9$$

Therefore $x = 14$
We can check by substituting 14 for x.

Equations which we solve at the same time in order to find two unknown values are called simultaneous equations.
An expression which contains a square as the highest power of any letter (x^2,y^2, etc) is called a quadratic. If we say that such an expression is equal to some value, the resulting equation is known as a quadratic equation.

EXAMPLE
Solve $x^2 - 5x + 6 = 0$.
By factorising we get $(x - 2)(x - 3) = 0$.
Therefore, either $x - 2 = 0$,
so $x = 2$,
or $x - 3 = 3$.
Therefore, $x = 2$ or 3.
These values for x are the roots of the equation.

Task 1

1) Find the number when seven times the number is four less than sixty-seven.

2) Find the number when twenty-eight is one more that three quarters of the number.

3) Find the number when five plus three times the number equals forty-one.

4) Three consecutive odd numbers add up to twenty-seven. What are they?

5) Two consecutive even numbers add up to thirty. What are they?

Section 2 Formulae

When we have solved a particular problem, we can often reduce the method of solving it to a fixed pattern and write down this pattern as a formula which can be used for similar problems.
For example, the statement "average speed is equal to the distance covered divided by the total time taken" can be written as the formula :

$$S = \frac{D}{T}$$

Often we will need to change the subject of a formula.
For example, from Boyle's law, we have the formula

$$P = \frac{k}{V}$$

We can change the subject of the formula to V, and the result is

$$V = \frac{k}{P}$$

Task 2

Read out the following formulae :

1) $C = \pi d$
2) $2S = U + V$
3) $x = a^n - b^2$
4) $V = u + at$
5) $I = \frac{PRT}{100}$
6) $F = \frac{9}{5}C + 32$
7) $V = \pi r^2 h$
8) $F = \frac{MV^2}{R}$
9) $E = mc^2$
10) $A = \pi r^2$

Task 3

Change the subjects of the above formulae as follows :

1) $d =$
2) $V =$
3) $b =$
4) $u =$
5) $P =$
6) $C =$
7) $h =$
8) $R =$
9) $m =$
10) $\pi =$

Task 4

Solve the following :

1) b plus eight equals eleven.
2) seven b equals forty-two.
3) two x equals one.
4) three y plus nine equals twenty-seven.
5) four y minus eleven equals y plus one.
6) seven b equals sixteen minus three b.
7) five c plus six equals two c plus twenty-four.
8) twelve plus four a equals seven a minus twenty-one.
9) twelve minus two b equals four b plus thirty-six.
10) three x plus five equals two x.

Task 5 Vocabulary practice

Fill in the blank spaces in the following sentences :

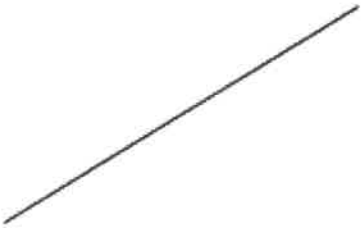
1) When we have _____ an equation, we should _____ our answer.
2) The answer is checked by _____ it for the letter in the original equation.
3) If our answer _____ the equation, it is correct.
4) We can use one equation to help us solve another equation. These are called _____ equations.
5) _____ are fixed equations which can be applied in certain regular situations.

UNIT 6 Lines and angles

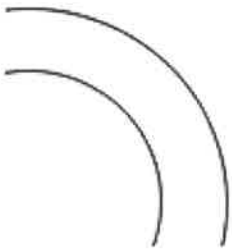
Section 1 Lines

1) This line is horizontal.

2) This line is vertical.

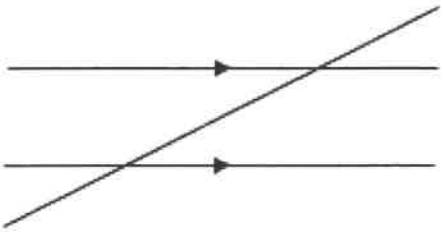


3) This line is oblique.

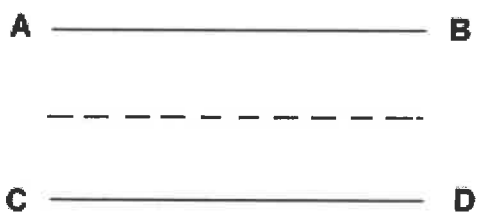


4) These lines are curved.

5) These two lines are parallel. They are equidistant at all points.



6) A straight line drawn across a set of two or more parallel lines is called a transversal.

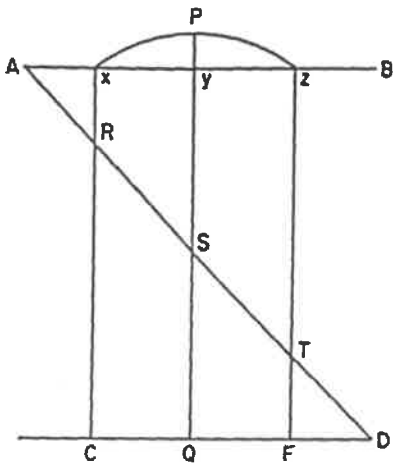


7) The broken line marks the locus of a point equidistant from AB and CD. The locus of a point is the path traced by that point when it moves in accordance with a given law.

Task 1

Look at the figure and say which lines are:

1) vertical
2) transversal
3) parallel
4) oblique
5) horizontal
6) curved

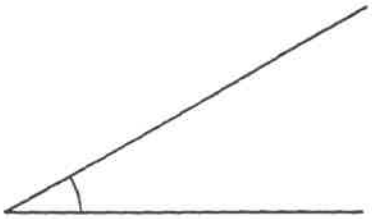


Task 2

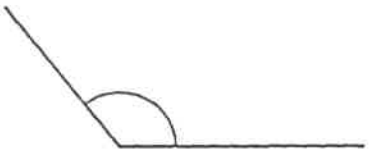
Using the words you have learned, describe the following mathematical symbols.

1) the plus symbol
2) the minus symbol
3) the multiplication symbol
4) the equals symbol
5) the pi symbol

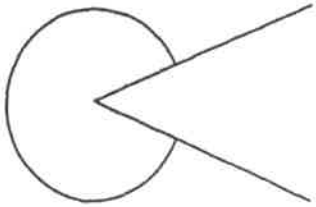
Section 2 Angles



1) These two lines meet at an angle. This angle is less than 90° (ninety degrees). It is an acute angle.

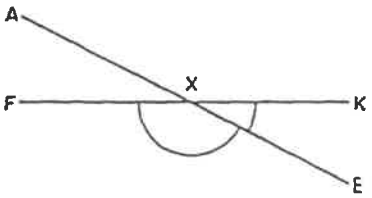


2) This is an obtuse angle.

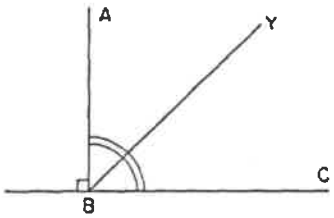


3) This is a reflex angle.

4) These two lines meet at an angle of 90°. They form a right angle. The two lines are perpendicular to each other.

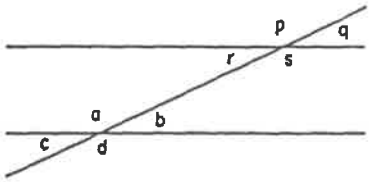


5) Lines FK and AB intersect at point X. The angles FXB and BXK are next to each other, or adjacent.
The sum of these angles is 180°. They are supplementary angles.



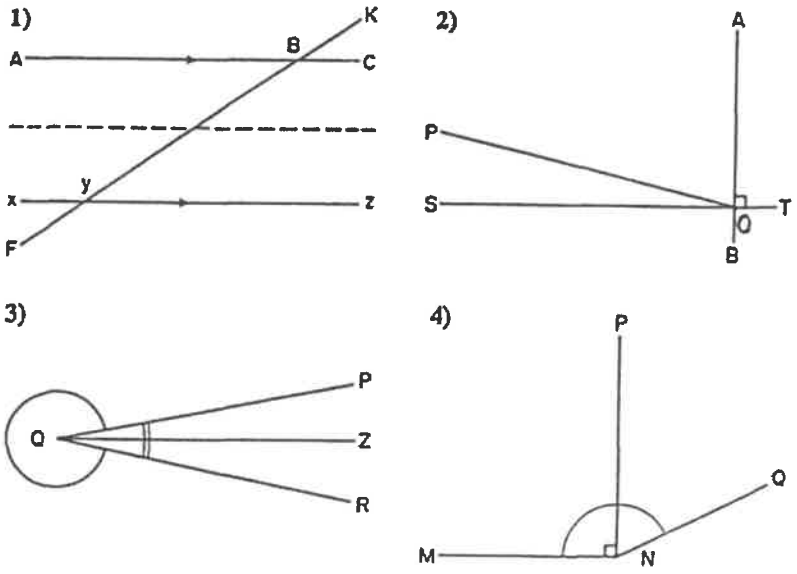
6) Angles ABY and YBC are equal.
Line BY bisects angle ABC.
BY is the bisector of angle ABC.
The sum of angles ABY and YBC is 90°. They are complementary angles.

7) This figure shows a transversal line drawn across two parallel lines.
Angles r and q are equal (opposite angles).
Angles b and q are equal (corresponding angles).
Angles b and r are equal (alternate angles).



Task 3

Describe the lines and angles in the following figures.

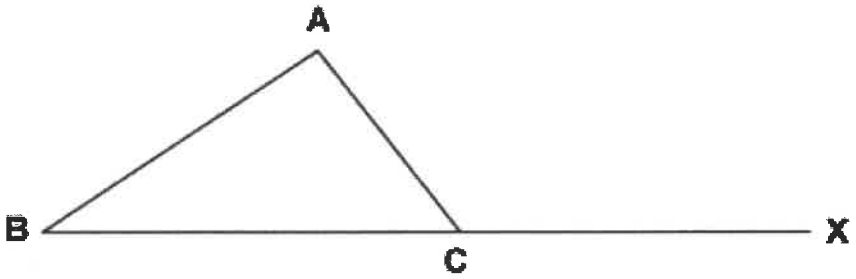


UNIT 7 Two-dimensional figures

Reading 1

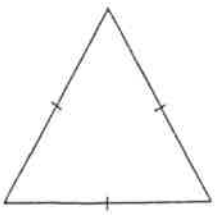
Section 1 The triangle

A triangle is a three-sided figure. The three sides of a triangle meet at points called vertices (singular vertex). The vertex at the top of a triangle may be called the apex, and the line at the bottom may be called the base.

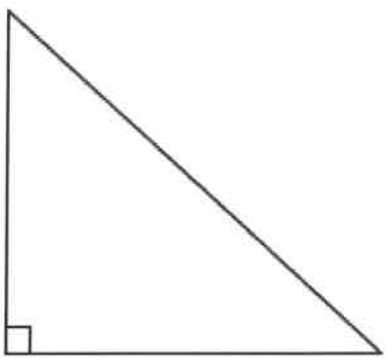


1) In triangle ABC, line BC is continued to point X. ACB is an interior angle, and ACX is an exterior angle.

2) This is an isosceles triangle.

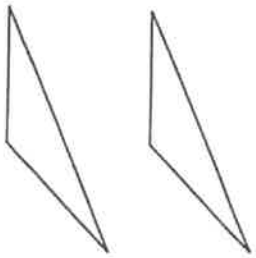


3) This is an equilateral triangle.

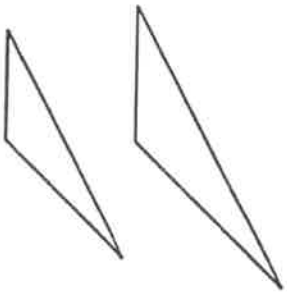


4) This is a right angled triangle. In a right angled triangle the side opposite the right angle is called the hypotenuse. The theorem of Pythagoras states : "In a right angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides".

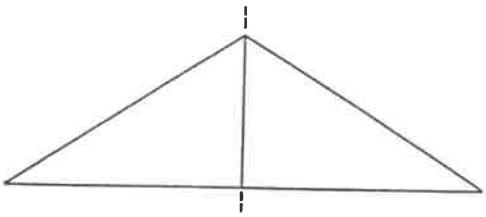
Section 2 The triangle - congruence, similarity and symmetry



1) If the following parts of two triangles are equal,
a) two sides and the included angle; or
b) a right angle, hypotenuse and side; or
c) two angles and a corresponding side;
then the two triangles are congruent.



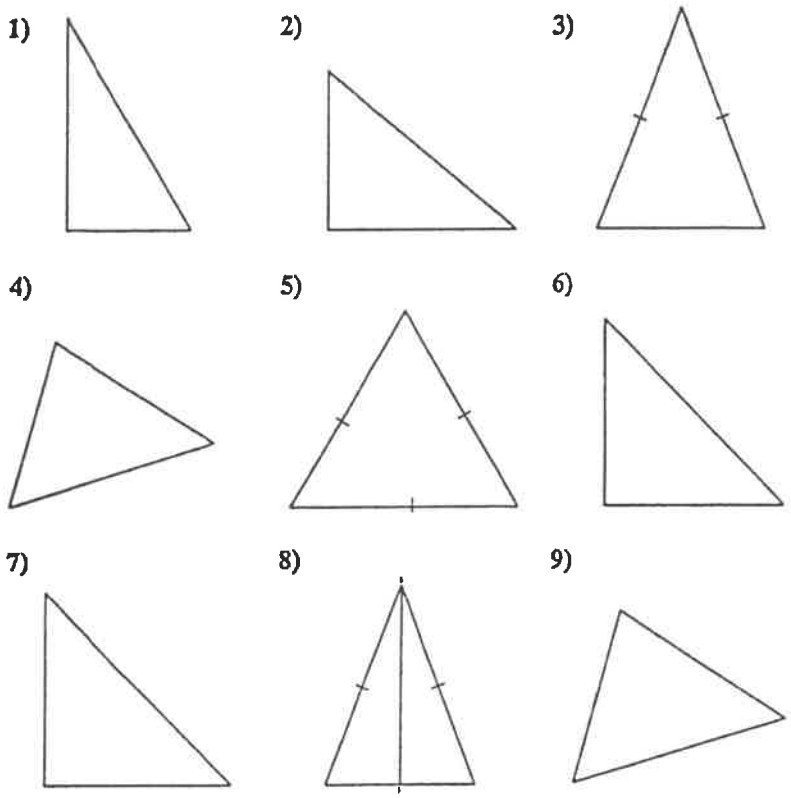
2) If two triangles have their corresponding angles equal, they are similar.



3) These two triangles are on either side of an axis of symmetry (or centre line). They are symmetrical triangles.

Task 1

Describe each triangle, and use your ruler to discover any relationships between the triangles (i.e. symmetry, similarity or congruence).



Task 2 Vocabulary practice

1) If each of the angles in a triangle is equal to 60°, the triangle is called _____.

2) A line which meets another _____ at 90° is called a _____ line.

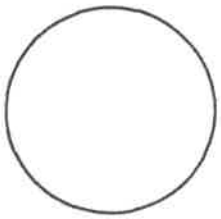
3) If two angles of a triangle are equal to 45°, the triangle is called a _____ _____ _____ triangle.

4) If we _____ a right angle, we have two _____ angles of 45°.

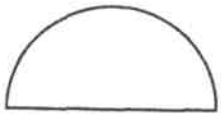
5) Each triangle has three points, or _____.

Reading 2

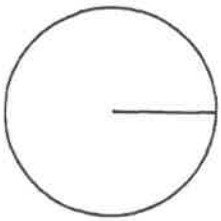
The circle



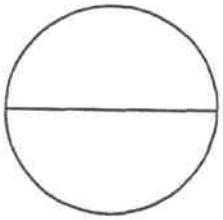
1) This is a circle.
The centre of a circle is called its point of origin.
The distance around a circle is called its circumference.



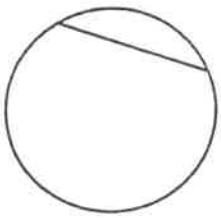
2) A half circle is called a semi-circle.



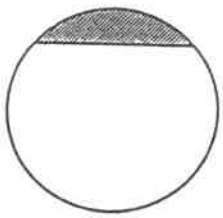
3) The line drawn from the point of origin to the circumference is called the radius (plural : radii).



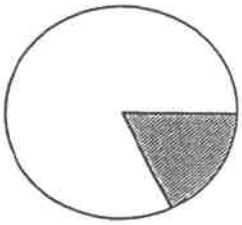
4) The line drawn from one side of the circle to the other, passing through the point of origin, is called the diameter.



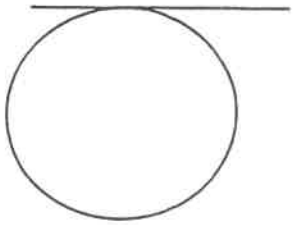
5) A part of the circumference of a circle is called an arc.
A straight line joining the ends of an arc is called a chord.



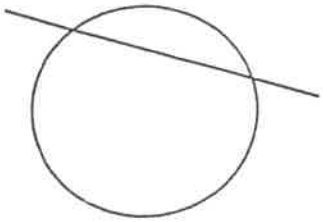
6) A part of a circle enclosed by an arc and a chord is called a segment.



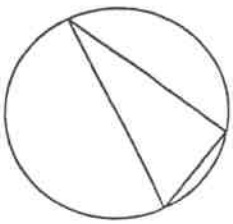
7) A part of a circle enclosed by two radii and an arc is called a sector.



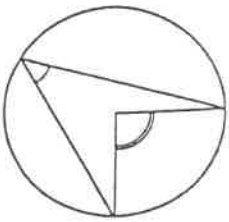
8) A line meeting the circumference but which (when produced) does not intersect it is called a tangent.



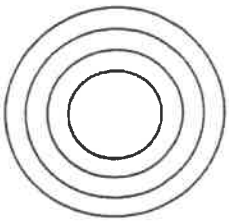
9) A line which intersects the circumference in two places is called a secant.



10) A circle which passes through the vertices of a triangle is called the circumference of the triangle, and its centre is called its circumcentre. The circle is circumscribed around the triangle.



11) The angle subtended at the centre by an arc of a circle is equal to twice the angle subtended by that arc at the circumference.

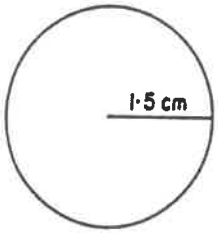


12) These circles have the same point of origin. They are concentric.

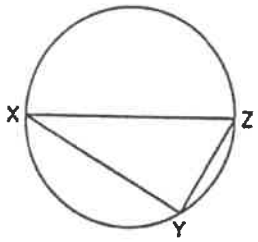
Task 3

Use the words you have learned to give information about the figures below.

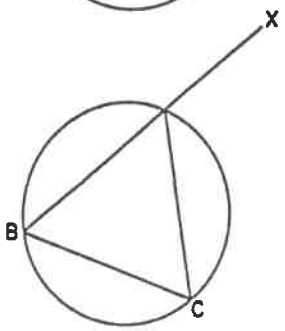
1)



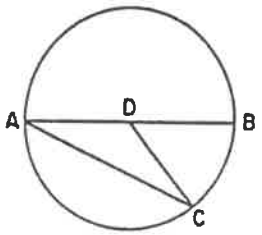
2)



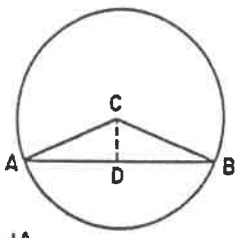
3)



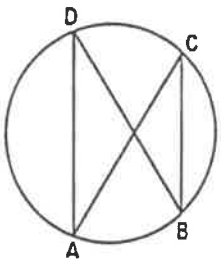
4)



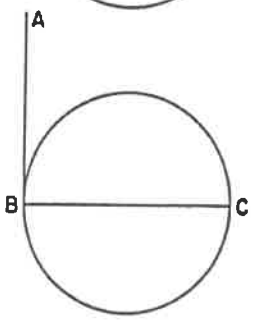
5)



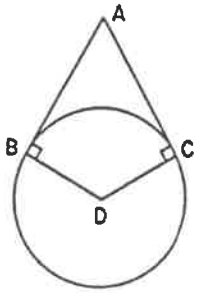
6)



7)



8)



Task 4

1) A circle has a radius of 3 centimetres. Calculate
   a) the diameter
   b) the circumference.
2) The circumference of a circle is approximately 15.7 centimetres. Calculate
   a) the approximate radius
   b) the approximate diameter.
3) What is the angle subtended by a semi-circle equal to?
4) The angle subtended by an arc at the circumference is 35°.
   a) What is the angle subtended by the same arc at the point of origin?
   b) How can you calculate the answer to a)?
5) If two chords of a circle, AB and CD, intersect at O, what is the relationship between AO × OB and CO × OD?

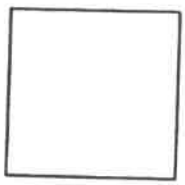
Task 5 Vocabulary practice

1) If we draw the _____ of a circle, the line divides the circle into two equal _____.
2) _____ circles are circles which have the same _____ of _____.
3) A semi-circle _____ an angle of 90° at the _____.
4) A triangle has been _____ if a circle passes through its _____.
5) A _____ is the area enclosed by an arc and two _____, while a _____ is the area enclosed by an arc and a _____.
6) If a line passes through a circle and intersects the circumference, it is called a _____, but a _____ meets the circumference without intersecting it.

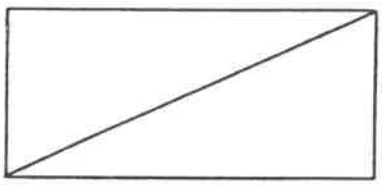
Reading

More 2-dimensional figures

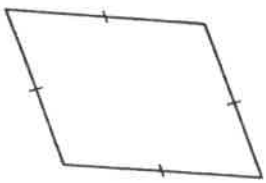
A line is **1-dimensional**. Triangles and circles are **2-dimensional**. Here are some more 2-dimensional figures.



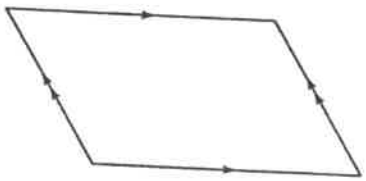
a **square**, **square**



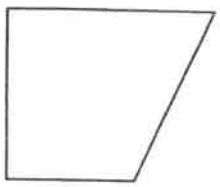
a **rectangle**, **rectangular**
The line drawn from one corner to the opposite corner is called the **diagonal**.



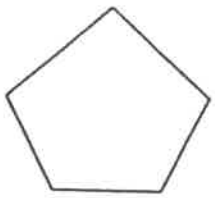
a **rhombus** (Pl. rhombuses, rhombi)
rhombic, **rhomboid**



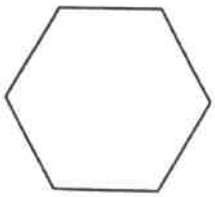
a **parallelogram** or a **rhomboid**
rhomboidal



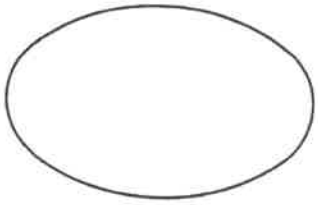
a **quadrilateral** or a **quadrangle**
quadrilateral



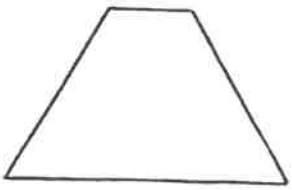
a **pentagon**, **pentagonal**



a **hexagon, hexagonal**



an **ellipse, elliptical**



a **trapezium, trapezoidal**

a **spiral, spiral**

NOTE 1 : A figure with many or an unspecified number of sides is called a **polygon.**
NOTE 2 : The sum of the sides of a two-dimensional figure is called a **perimeter.**

Task 6

Explain the following terms in English:

1) the Pentagon
2) the Parallelogram of Forces Rule
3) the elliptical galaxy (in contrast to the spiral galaxy)

UNIT 8 Three-dimensional figures

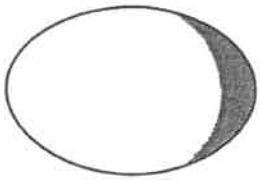
Tuning-in

Task 1

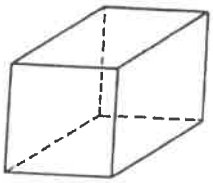
What is the difference between a line, a triangle, and a pyramid in terms of their dimensional characteristics?

The following figures are **3-dimensional**.

a **sphere, spherical**



an **ellipsoid, ellipsoid**

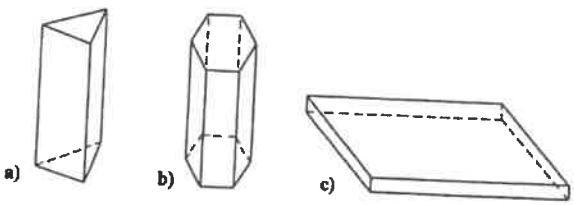


a **cube, cubic**

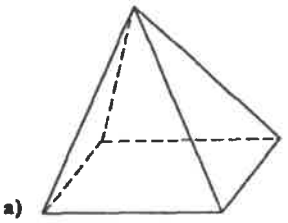
a **helix** (Pl.helices, helixes), **helical**

a **prism, prismatic**

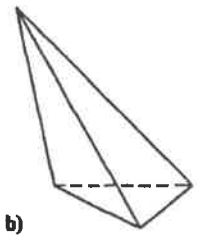
a) **a triangular prism**
b) **a hexagonal prism**
c) **a rectangular prism**



a **pyramid, pyramidal**



a) a **right** square **pyramid**



b) an **oblique** triangular **pyramid**

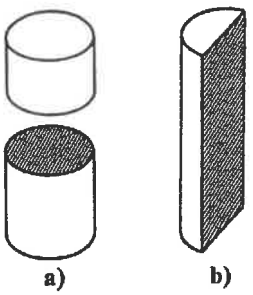
a **cone, conical**

a **cylinder, cylindrical**

These figures show **cross-sections** of a cylinder.

a) is a **transverse** section (circular).

b) is a **longitudinal** section (rectangular).



Task 2 Vocabulary practice

1) A _____ is a _____ with six sides.

2) A four-sided figure is called a _____.

3) A shape with five sides is a _____ shape.

4) A four-sided figure with two sides parallel is called a _____.

5) A rhomboid has two _____ and two _____ angles.

6) The _____ of the _____ angles of a polygon is equal to 360°.

7) A _____ may be called an equilateral rectangle.

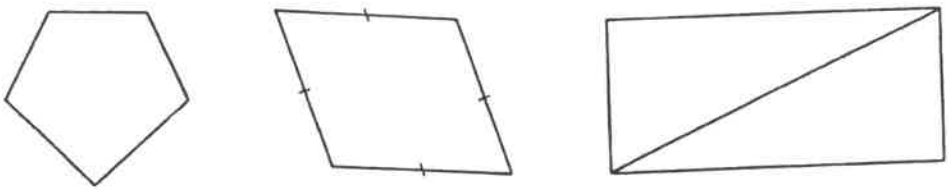
8) If two _____ of a parallelogram are vertical, the other two are _____.

9) A _____ which has length and width is _____-_____.

10) A figure with four equal _____ but no right angles is called a _____.

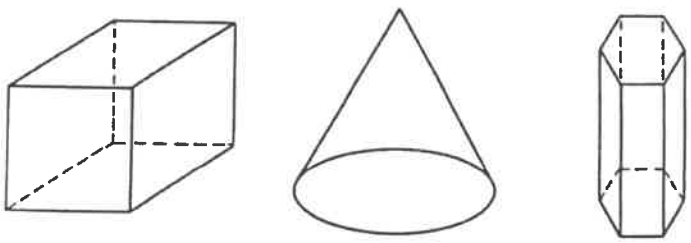
Task 3

Name the following shapes and describe their lines and angles.



Task 4

Identify the following shapes and their sections.



Task 5

Draw the following sections :

1) an oblique section of a cone.

2) a longitudinal section of a right triangular pyramid.

3) a transversal section of a cylinder.