

UNIT 1

Introduction to Mathematics and numbers

Tuning-in

Task 1

Try to answer the following questions:

- 1) What are numbers?
- 2) What are integers?
- 3) What are digits?

Note: the word digit comes from a Latin word meaning finger.

- 4) What types of numbers are you familiar with?
- 5) What is the difference between 3,142 and 3,142?

Task 2

Explain the difference between the following pairs of words:

- 1) cardinal - ordinal numbers
- 2) odd - even numbers
- 3) common fractions - decimal fractions

Task 3

Define the following terms in mathematics and give examples: a prime number, a three-digit number, a number line.

Note: a two-, three-, four-, etc. digit number, but numbers from 0-9 are known as one- or single-digit numbers.

Reading 1

Task 4

Read the following text to check some of your answers to Tasks 1-3.

Mathematics and numbers

It is said that mathematics is the basis of all other sciences, and that arithmetic is the basis of mathematics. Numbers consist of whole numbers (integers) which are formed by digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 and by combinations of them. E.g. 247 - two hundred and forty seven - is a number formed by three digits. Parts of numbers smaller than 1 are sometimes expressed in terms of fractions, but in scientific usage they are given as decimals. This is because it is easier to perform the various mathematical operations if decimals are used instead of fractions.

The decimal is always represented by point, and not by comma which is reserved for thousands. The main operations are: addition, subtraction, multiplication and division, taking a root and raising to the power. The decimal, or ten-scale system, is used for scientific purposes throughout the world. The other scale in general use nowadays is the binary, or two-scale, in which numbers are expressed by combinations of only two digits, 0 and 1. Thus, in the binary scale, 2 is expressed as 010, 3 as 011, 4 is represented as 100, etc. This scale is perfectly adapted to the "on-off" pulses of electricity, so it is widely used in electronic computers.

Other branches of mathematics such as algebra and geometry are also extensively used in many sciences and even in some areas of philosophy. Finally, a knowledge of statistics is required by every type of scientist for the analysis of data.

Language study 1

Mathematical symbols

Scientific texts can easily be recognised by the use of diagrams, symbols, formulae and equations. They are the same in all written languages but are pronounced differently.

Symbols	How the symbol is read
+	plus; positive
-	minus; negative
$ab, a \cdot b, a \times b$	ab : a times b , a multiplied by b
$a/b, a : b, a \div b$	a over b , a divided by b , the ratio of a to b
=	equals, is equal to, is
\equiv	is identical with, is equivalent to
\neq	does not equal, is not equal to
$>$	is greater than
$<$	is less than
\gg	is substantially greater than, is much greater than
\ll	is substantially less than, is much less than
$>$	is greater than or equal to
\geq	is less than or equal to
a^2	a squared
b^3	b cubed
12^5	twelve to the power of five, the fifth power of 12
$\sqrt{9}$	the square root of nine
$\sqrt[3]{10}$	the cube root of ten
$\sqrt[n]{a}$	the n th root of a
()	round brackets, parentheses
[]	square brackets, brackets
{ }	curly brackets, braces
$\log_a x $	logarithm of x to the base a
$n!$	factorial n , n factorial
a'	a prime
a''	a double prime, a second
a_n	a sub n , a subscript n
\int	integral

Task 5

How would you pronounce the following?

- 1) $3^2 = 9$
- 2) $(a + b)^4$
- 3) $\sqrt[3]{81} \gg \sqrt[3]{8}$
- 4) b_n
- 5) $b \leq 8$
- 6) $5^3 - \sqrt[3]{27} = 122$
- 7) $(a + b) - (c + d)$
- 8) $\log_2 |2|a$
- 9) $12x : 4 = 3x$
- 10) $a^4 \cdot a^3 = a^7$

Language study 2

Irregular plurals

Many scientific and technical terms which have come into English from other languages like Latin and Greek form their plurals in a completely different way. Therefore, they are often referred to as irregular plurals. Some of them are given below.

radius, radii
 nucleus, nuclei
 locus, loci
 axis, axes
 analysis, analyses
 basis, bases
 ellipse, ellipses
 hypothesis, hypotheses
 parenthesis, parentheses
 matrix, matrices or matrixes
 index, indices or indexes
 appendix, appendices or appendixes
 vertex, vertices
 apex, apices or apexes
 criterion, criteria
 phenomenon, phenomena
 formula, formulae or formulas
 abscissa, abscissae or abscissas
 maximum, maxima
 minimum, minima

Task 6

Give the Croatian equivalents of the above nouns.

Word study

Task 7

Arithmetical operations on numbers include addition, subtraction, division and multiplication.

One number may be added to another. The result is called the sum. The sum of 9 and 14 is 23, i.e. 9 plus 14 equals 23, or 9 plus 14 is equal to 23.

Make similar statements using these words:

- 1) subtracted/difference

- 2) multiplied/product
- 3) divided/quotient

Pay attention to the necessary preposition.

Task 8

An integer is even if it is divisible by 2.

An integer is odd if it is not divisible by 2.

An integer is divisible by 3 if the sum of its digits is divisible by 3.

Now make similar statements about the divisibility of integers by:

- 1) 10
- 2) 9
- 3) 4
- 4) 8
- 5) 5
- 6) 6
- 7) 11

Reading 2

Task 9

Read the texts and identify the types of numbers listed in the two texts. Give their definitions.

The number system

The set of positive and negative integers consists of all the natural numbers 1, 2, 3, 4..., plus the same numbers preceded by the minus sign, -1, -2, -3, We can represent any of these numbers on the number line. We can also represent fractions of numbers, e.g. 1.5 , $\frac{2}{3}$, -3.4 etc., on the number line. The rational numbers are composed of both the integers (or whole numbers) and the non-integers (or fractions). All rational numbers may be represented as a fraction where both the denominator and the numerator are integers, whereas irrational numbers cannot be expressed in this way. Irrational numbers include numbers like π (3.14159), $\sqrt{2}$ (1.41421), $\sqrt[3]{5}$ (1.70997) and so on. All these numbers, both rational and irrational, make up the set of real numbers, and may be represented as points on a number line. Imaginary numbers, on the other hand, cannot be represented as points on a number line. They include numbers such as $\sqrt{-1}$, which is usually expressed by the symbol i . Finally, a complex number is a number which contains both a real number and an imaginary number, for example $6 + \sqrt{-4}$.

Sets of numbers

Several sets of numbers are used frequently in mathematics and the use of standard abbreviations or symbols referring to them enables us to save time and space. Capital letters are usually used for this notation. The set of natural numbers is denoted by \mathbb{N} . \mathbb{Z} denotes the set of all integers. \mathbb{R} represents all real numbers and \mathbb{Q} all rational numbers.

Note that \mathbb{Z}^+ means all positive integers, \mathbb{R}^- all negative real numbers, and so on. \mathbb{Z}^+ differs from \mathbb{N} in that \mathbb{Z}^+ consists of all positive integers, whereas \mathbb{N} consists of all non-negative integers and therefore includes the element zero in addition to the elements of \mathbb{Z}^+ . We can see from this that a set which contains only the element zero is not the same as the empty set, which contains no elements. Thus $\mathbb{N} - \mathbb{Z}^+ = \{0\}$, but $\mathbb{Z}^+ \cap \mathbb{Z}^- = \emptyset$.

Some other sets are also referred to in the form of abbreviations. We use a capital U to refer to the universal set, while the empty set is denoted by a symbol which consists of a zero bisected by an oblique line. This may also be read as the null or void set. For example, if two sets, A and B are disjoint (that is, they have no elements in common), then $A \cap B = \emptyset$.

Task 10

Look at this set of numbers:

2, 3, 5, 7, 11, 13, 17, 19, 23...

- 1) Can you continue this set?
- 2) Which numbers does it consist of?

Task 11

List the following sets:

- 1) the set of whole numbers less than 5
- 2) the set of natural numbers less than 5
- 3) the set of even numbers greater than 7 and less than 15
- 4) the set of two-digit numbers less than 50 with both digits the same
- 5) the set of two-digit numbers the sum of whose digits is 4.

Note: When you specify a set of numbers, you list the numbers inside braces {}.

Task 12

a) Say whether the following statements are true or false. Correct the false statements.

- 1) If m and n are integers, $\frac{m}{n}$ is a rational member.
- 2) The set of irrational numbers includes negative integers.
- 3) Neither irrational numbers nor complex numbers may be represented as points on a number line.
- 4) The symbol i represents a complex number.
- 5) A complex number consists of at least two parts.

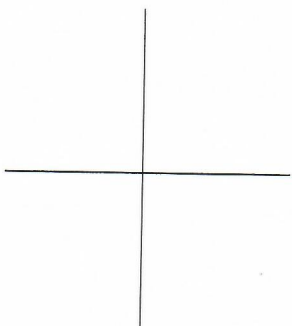
b) Describe the following numbers:

-4, $\frac{3}{5}$, $\sqrt{2}$, 1.41, $\sqrt[3]{27}$

Example: 2.5 positive rational non-integer.

Task 13

a) Label this diagram as you read the passage:



This diagram consists of two lines. The horizontal line is called the x -axis; the vertical is called the y -axis. They intersect at the origin O . Together, these two axes are called the co-ordinate axes. The axes separate the diagram into four quadrants. The top right-hand quadrant is known as the first quadrant, the top left the second, the bottom left the third, the bottom right the fourth. Starting from the origin, label the points to the right on the x -axis 1, 2, 3, 4, etc... and to the left -1, -2, -3, and so on. On the y -axis, starting from the origin, label the points above the origin 1, 2, 3, 4 ... and the points below

the origin -1, -2, -3, etc. Now any point in the plane may be represented in the relation to the two axes by two numbers. For example, the point P is represented by two numbers, (4, 2). These numbers are called the co-ordinates of point P. The x-co-ordinate, 4 is called the abscissa of P. The y-co-ordinate, 2 is called the ordinate of P. The system is known as the Cartesian co-ordinate system.

- b) Say whether the following statements are true or false. Correct the false statements.
- 1) The abscissa of the point Q is 3.
 - 2) The co-ordinates of the point R are (-1, 2).
 - 3) The co-ordinates of the point P are (2, 4).
 - 4) The two axes divide the plane into four quadrants.

UNIT 2 Fractions

Tuning-in

Task 1

Answer the following questions:

- 1) How can you define fractions?
- 2) How many types of fractions are you familiar with?
- 3) How often do you use fractions in everyday life? On which occasions?

Reading 1

Section 1 Vulgar or common (proper) fractions

$\frac{4}{5}$ (four fifths or four over five) is a fraction.

In this fraction, 4 is the numerator and 5 is the denominator.

$\frac{9}{16}$ is an improper fraction.

$4\frac{1}{4}$ is a mixed number.

To add or subtract vulgar fractions, we must express them in terms of the lowest common denominator. For example, in expression $\frac{2}{3} - \frac{1}{5} = \frac{7}{15}$ the lowest common denominator is 15.

To multiply or divide vulgar fractions e.g. $2\frac{5}{8} \times 2\frac{2}{3} \times 1\frac{3}{10}$ we must first change the mixed numbers to improper fractions $\frac{21}{8} \times \frac{8}{3} \times \frac{13}{10}$ and then cancel where possible

$\frac{7}{1} \times \frac{1}{1} \times \frac{13}{10}$. Then we multiply the numerators and denominators and express the result as a mixed number $= \frac{91}{10} = 9\frac{1}{10}$

Section 2 Decimal fractions

To write a decimal fraction we use a decimal point.

For example, if we convert $2\frac{1}{4}$ into a decimal fraction, the result is 2.25 (two point two five).

If we convert $\frac{2}{3}$ into a decimal fraction, the result is 0.6 (nought point six recurring).

Note that $17 : 3 = 5.6$ or 5.67 correct to two decimal places.

Note also that π is equal to 3.142 correct to four significant figures.

Task 2 Vocabulary practice

Using single words, fill in the blank spaces in the following sentences :

- 1) In the vulgar fraction seven ninths ----- is the numerator and ----- is the -----.

- 2) To ----- a vulgar fraction to a decimal fraction, we simply ----- the numerator by the denominator.
- 3) The ----- of two thirds and a half is six.
- 4) An integer plus a fraction makes a -----.
- 5) An improper fraction exists when the ----- is greater than the -----.
- 6) To multiply a decimal fraction by ten, we simply move the ----- one place to the right.
- 7) 57.074 correct to ----- is 57.1.
- 8) To add or subtract vulgar fractions, we must ----- them ----- their lowest common denominator.
- 9) To divide a decimal fraction by ----- we simply move the decimal point one ----- to the -----.
- 10) $\frac{5}{2} \times \frac{2}{5}$ becomes $\frac{5}{5}$ if we ----- the two's.

Task 3 Vocabulary practice

Using single words, fill in the blank spaces in the following sentences :

- 1) The ----- of three and four is twelve.
- 2) The operation which uses the symbol : is called -----.
- 3) Forty-eight ----- thirty-six equals twelve.
- 4) The result of a division problem is called the -----.
- 5) A whole number is also known as an -----.
- 6) Any number consists of combinations of -----.
- 7) Eighteen subtracted ----- twenty equals -----.
- 8) Three multiplied ----- five equals -----.
- 9) When we ----- two quantities, for example seven plus twelve, the answer (nineteen) is called the -----.
- 10) The product is the result when one quantity is ----- another.

Task 4

Fill in the blank spaces in the following sentences and answer the respective questions:

A number such as $\frac{3}{5}$ is called a ----- . A fraction comprises two parts, a ----- and a ----- . The denominator is the number ----- the line.

1) What is the numerator?

2) What are the numerator and denominator separated by?

If the numerator is ----- than the denominator, the fraction is known as a proper ----- . If the denominator is less than the -----, the fraction is known as an ----- fraction. In the fraction $\frac{102}{153}$ both the denominator and the numerator may be ----- by the same number (-----) to give $\frac{2}{3}$. This is called ----- or reducing the fraction.

3) Can the following fractions be reduced?
 $\frac{28}{70}$ $\frac{40}{64}$ $\frac{41}{105}$

Reading 2

Ratio, proportion and percentage

Section 1 Ratio and proportion

The ratio of two quantities is the magnitude of one quantity relative to the other. Division of quantity a by quantity b gives the ratio $\frac{a}{b}$, which can also be written as $a : b$ and is read as 'the ratio of a to b '. For example, the ratio of boys to girls in a particular school is 3:2. If the school has 250 pupils, then we can see that $\frac{3}{5}$ of these are boys and $\frac{2}{5}$ girls, i.e. there are 150 boys and 100 girls.

When we build a model ship, we make it to scale. For example, if a model is built to a scale 1:30 (one to thirty), this means that 10 centimeters on the model represents 300 centimeters on the ship itself. The scale of a map shows the ratio of the distance on the map to the distance on the area covered by the map. On a map this ratio is called the representative fraction.

Relative sizes of more than two quantities may be expressed by ratio. For example, the ratio of AB:BC:AC in a certain triangle $\triangle ABC$ is 3:4:6. Hence we can see that AC is twice as long as AB. Any triangle which is similar to triangle $\triangle ABC$ has sides in exactly the same ratio.

When the ratio of one pair of quantities is equal to the ratio of another pair of quantities, the two pairs are said to be in proportion. If we say that a, b, c are in proportion, we mean that $\frac{a}{b} = \frac{c}{d}$. A property of this proportion is that the reciprocals are also in proportion. Moreover, the ratio of the numerators is equal to the ratio of the denominators.

$y \propto x$ The ratio between y and x is a constant.
 Y is directly proportional to x .
 We say that y varies directly with x .

$y \propto \frac{1}{x}$ y is directly proportional to the reciprocal of x .
 We say that y is inversely proportional to x
 and that y varies inversely with x .

$y \propto xz$ y is directly proportional to the product of x and z .
 We say that y is jointly proportional to x and z ,
 and that y varies jointly with x and z .

Task 5

Answer the following questions or provide solutions to the given mathematical problems.

- 1) Which fraction with a denominator of sixteen is in proportion to one over four?
- 2) If a plan is drawn to a scale of 1:50 (one to fifty), what is the actual measurement which is shown on the plan as four centimeters?
- 3) Divide one hundred and forty sheep into two groups in the ratio of 3:4.
- 4) The scale of a map is five centimeters to one kilometre. What is the representative fraction of the map?
- 5) On the same map, what length will represent nine kilometres?
- 6) Divide thirty six pounds into three parts in the ratio 6:5:1.
- 7) Five families have a total of 100 sheep. How many sheep will six families have if the numbers are in proportion?
- 8) A concrete mix of cement, sand and gravel is made in the ratio of 2:5:8. What is the weight of each part in thirty tonnes of concrete?
- 9) In a class of students the ratio of success to failure in an examination was 9:2. If eighteen students passed the examination, how many failed?
- 10) If ten litres of oil weigh eight kilograms, and a litre of water weighs one kilogram, what is the ratio of the relative density of oil and water?

Task 6

Describe the relationship between the following quantities, where k is a constant:

1) $y = \frac{k}{x}$

2) $y = kxz$

3) $y : x = k$

4) $y = kx$

5) $y : \frac{1}{x} = k$

Task 7

The volume of a gas is inversely proportional to its pressure. The smaller the volume, the higher the pressure.
 Now make similar sentences about the following:

- 1) the density and pressure of a gas (for a constant temperature)
- 2) the volume and temperature of a gas (for a constant pressure)
- 3) the velocity of a falling body and the time it has been falling
- 4) acceleration and mass for a constant force

Task 8

Say whether the following statements are true or false. Correct the false statements:

- 1) In a test a student scored 30 out of 100. This gives a ratio of 3:10 for correct answers to the incorrect ones.
- 2) The length of the shortest side of a triangle similar to triangle ABC is 12 cm. The other sides are therefore 16 and 24 cm long.
- 3) Any triangle with sides in the ratio 2:3:6 is a right-angled triangle.
- 4) If a , b , c are in proportion, then $ad = bc$.

Section 2 Percentage

36% (thirty six percent) is really a Fraction with a numerator of thirty six and a denominator of one hundred.

The fraction $\frac{8}{40}$ expressed as a percentage is 20%.

If a number is decreased by 10% the ratio of the new number to the old number is 90:100.

If a number is increased by 10% the ratio of the new number to the old number is 110:100.

If we borrow a sum of money at a rate of interest of 10% we must pay back the money in the same proportion.

If a student scores 81% in one exam and 87% in the next, his average (or mean) percentage is 84%.