

UNIT 3 Powers and roots

Reading

Section 1 Powers

When we write 4^2 (four squared), or x^2 (x squared), the 2 is called the power or index.

$a \times a^2$ (a times a squared) is equal to a^3 (a cubed).

$a^3 \times a^2$ (a cubed times a squared) is equal to a^5 (a to the power of five or a to the fifth (power)).

In this example we simply add the indices or powers.

$(x^2)^3$ (x squared all cubed) is equal to x^6 (x to the power of six).

This example shows that to raise a power to a power, we multiply the indices.

Section 2 Roots

$\sqrt{64}$ means the square root of 64.

$\sqrt[3]{27}$ means the cube root of 27.

$\sqrt[5]{x}$ means the fifth root of x .

For example, if we wish to find the root in, for example,

$$\sqrt[n]{a^m}$$

we must divide the index by the root

$$= a^{m:n} = a^{m/n}$$

Section 3 Fractional and negative indices

$\sqrt[3]{x^2}$ is equal to $x^{\frac{2}{3}}$. Here the index ($\frac{2}{3}$) is a fraction and is therefore called a fractional index.

$x^2 : x^4 = x^{-2}$ (x to the [power of] minus 2) which is called a negative index.

Task 1 Vocabulary practice

Fill in the blank spaces in the following sentences :

- 1) Any number to the _____ of 0 (nought) is equal to _____.
- 2) To divide powers we _____ the _____.
- 3) To _____ a _____ to a power, we _____ the indices.
- 4) a to the _____ of five divided _____ a _____ equals a cubed.
- 5) The _____ of forty-nine is seven.

Task 2

How would you pronounce the following:

1) $a^2 + b^3$

2) x^4

3) $z^n = (x + y)^2$

4) $6y^2 : 3y^2$

5) $\sqrt{x^2}$

6) $\sqrt[3]{b^3}$

7) $\sqrt{16b^2}$

8) $\sqrt[3]{8x^6y^9}$

9) $9^{\frac{1}{2}}$

10) 2^{-3}

Task 3

Solve the following and read the whole procedure aloud:

1) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

2) $2^{-5} =$

3) $2^{\frac{3}{2}} =$

4) $2^3 + 2^4 =$

5) $(2^3)^2 =$

Language study**Word transformation**

In English a word that belongs to one part of speech can frequently be transformed with the addition of an ending to another part of speech. These transformations most frequently happen among nouns, verbs and adjectives. The simplest transformation is the transformation of an adjective into an adverb with the addition of the ending - *ly*.

Task 4

Give several examples to support this statement.

Sometimes no transformation of a word is visible and the word still changes from one part of speech into another. This 'invisible' transformation usually occurs between nouns and verbs.

Task 5

List several examples of such transformation.

Task 6

Give some examples of the words that represent the 'invisible' transformation between nouns and adjectives.

UNIT 4 Factors

Reading

Section 1 Arithmetical factors

If one number divides exactly into a second number, the first is a factor of the second, and the second is a multiple of the first.

The fraction $\frac{8}{24}$ is normally written as $\frac{1}{3}$. It is normal to express fractions in their lowest terms. Here 2, 4 and 8 are all factors of both the numerator and the denominator, but 8 is the highest common factor (H.C.F.). A factor which is also a prime number (1, 2, 3, 5, 7, 11 etc) is called a prime factor.

The smallest number which is exactly divisible by two or more numbers is called their lowest common multiple (L.C.M.). The L.C.M. of 24 and 36 is 72. The L.C.M. is also known as the least common multiple.

Task 1

Write down the answers to the following :

- 1) What are the prime factors of thirty-eight?
- 2) What is the highest common factor of eighteen and twenty-six?
- 3) What is the lowest common multiple of six and eight?
- 4) Express the fraction fourteen over twenty-one in its lowest terms.

Task 2

Ask and answer questions about the highest common factors and lowest common multiples of the following pairs of numbers :

- 1) 36, 42
- 2) 218, 78
- 3) 142, 82
- 4) $12xy, 3x^2$

Section 2 Algebraic factors

If the expression $3x(3x - 5)$ is expanded, we obtain the result $9x^2 - 15x$.

If the expression $9x^2 - 15x$ is factorised, we reverse the process and obtain the result $3x(3x - 5)$.

In an expression where it is difficult to discover the H.C.F., it helps to group the terms.

EXAMPLE

Factorise the following expression :

$$ax - ay + bx - by$$

a is a factor of the first two terms, and b is a factor of the second two terms. Thus, by grouping the terms, we obtain

$$a(x - y) + b(x - y)$$

Here $(x - y)$ is a common factor, so we factorise again to obtain the result :

$$(x - y)(a + b)$$

If an algebraic expression is made up of two terms, e.g. $(x + 3)$ or $(2y - 4)$, it is called a binomial.

An algebraic expression that is made up of three terms e.g. $12x^2 + 13x - 4$ is called a trinomial.

A trinomial is the product of two binomials;

e.g. $(a + 5)(a - 2) = a^2 + 3a - 10$

and so the factors of a trinomial can be expressed as two binomials.

Task 3

Expand the following :

- 1) Three x minus four all squared.
- 2) Two y plus nine all squared.
- 3) Five a minus four all squared.
- 4) Four plus two r all squared.
- 5) Four p minus two q all squared.

Task 4

Factorise the following :

- 1) x squared plus two xy plus y squared.
- 2) nine a squared plus eighteen ab plus nine b squared.
- 3) thirty-six minus sixteen a squared.
- 4) four minus b squared.
- 5) x squared minus y squared.

Task 5 Vocabulary practice

Fill in the blank spaces in the following sentences :

- 1) Twenty-three has only two _____, itself and _____, and is therefore a _____ number.
- 2) To express a fraction or algebraic expression in its _____, we must divide its terms by the _____.
- 3) The factors of a _____ can be expressed as the _____ of two binomials.
- 4) Twenty-four is the _____ of twelve and eight.

Task 6

Any integer may be represented as the product of prime numbers. For example, $150 = 2 \cdot 3 \cdot 5^2$. This is known as factorising a number. 20 can be factorised into $2^2 \cdot 5$.

Make similar statements about these numbers :

- 1) 16
- 2) 24
- 3) 36
- 4) 370

Word study**Verbs and related nouns***Task 7*

Each of the verbs in column A has a related noun in column B. Complete the blanks. For some counterparts more than one word is possible. Explain the difference. For example: express - expression

A Verbs	B Nouns
1 factorise	_____
2 _____	definition
3 multiply	_____
4 convert	_____
5 _____	abbreviation