Pijavskij algoritam (S. A. Pijavskij, *An algorithm for searching for a global minimum of a function*, USSR Computational Mathematics and Mathematical Physics, **12**(1972), 888–896)

Algorithm (Pijavskij's algorithm)

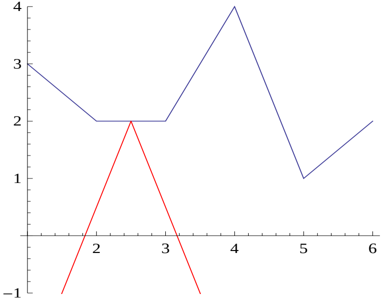
Input: $f : [a, b] \to \mathbb{R}, \quad L > 0$
Input: $u_0 \in [a, b]$; $n \geq 1$; $k = 0$;
{Input the function and the allowed number of iterations;}

1: Define $K(u; u_0) := f(u_0) - L|u - u_0|$ and $P_0(u) := K(u; u_0)$;
2: Determine $u_1 \in \underset{u \in [a,b]}{\operatorname{argmin}} P_0(u) \subseteq \{a, b\}$;
3: **while** $k = k + 1; k < n$, **do**
4: Define $K(u; u_k) := f(u_k) - L|u - u_k|$;
5: $P_k(u) := \max\limits_{i=0,1,\ldots,k} K(u; u_i) = \max\{K(u; u_k), P_{k-1}(u)\}$;
6: Determine $u_{k+1} \in \underset{u \in [a,b]}{\operatorname{argmin}} P_k(u)$;
7: **end while**

Output: $\{u_{k+1}, f(u_{k+1})\}$

Pijavskij's algorithm

- Izabrati $u_0 \in [a,b]$ i definirati $K(u;u_0) := f(u_0) - L|u-u_0|$; $\quad u_1 = \operatorname*{argmin}_{u\in[a,b]} K(u;u_0)$;
- Definirati $K(u;u_1) := f(u_1) - L|u-u_1|$ i
  $P_1(u) := \max_{i=0,1} K(u;u_i) = \max\{K(u;u_1), P_0(u)\}, \quad P_0(u) := K(u;u_0)$;
  $u_2 = \operatorname*{argmin}_{u\in[a,b]} P_1(u)$;
- Definirati $K(u;u_2) := f(u_2) - L|u-u_2|$ i
  $P_2(u) := \max_{i=0,1,2} K(u;u_i) = \max\{K(u;u_2), P_1(u)\}; \quad u_3 = \operatorname*{argmin}_{u\in[a,b]} P_2(u)$



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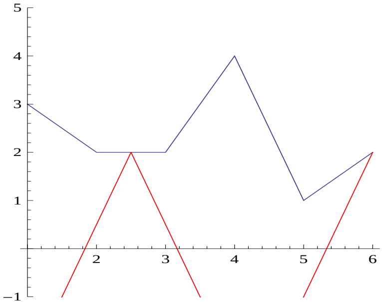

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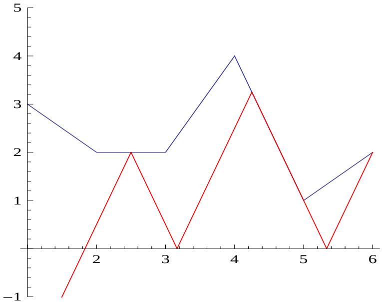

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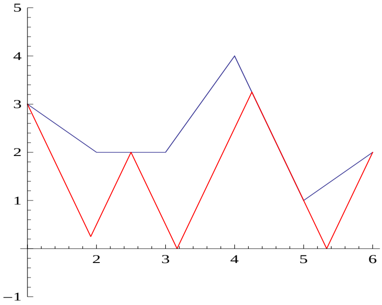

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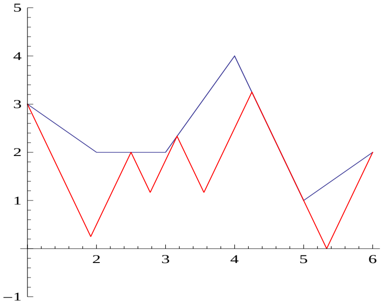

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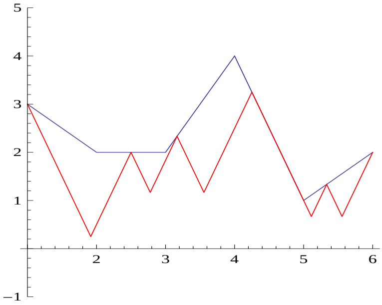

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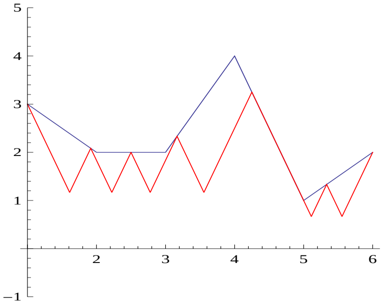

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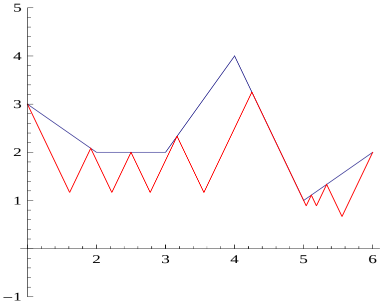

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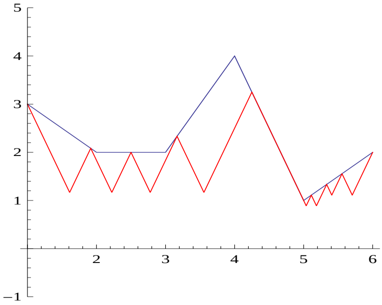

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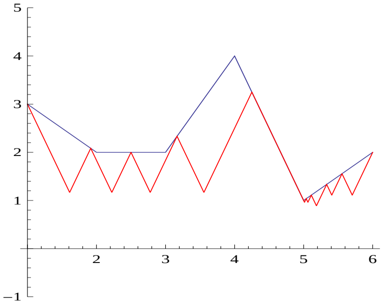

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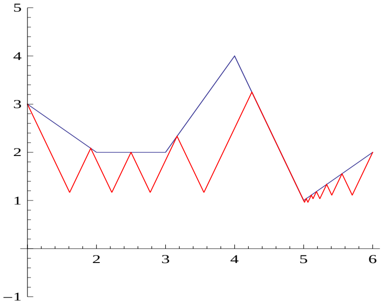


Svojstva metode slomljenih pravaca

Lemma 1

Let $f \in Lip_L[a, b]$, (u_n) a sequence of nods, and (P_n) the sequence of functions defined as in Pijavskij's algorithm. Then

(*i*) P_n *are Lipschitz-continuous piecewise linear functions whose parts belong to straight lines with coefficients* L *or* $-L$*, i.e.* $P_n \in Lip_L[a, b]$;

(*ii*) $P_n(u) \leq P_{n+1}(u)$ *for all* $u \in [a, b]$ *and all* $n = 0, 1, \ldots$

[monotonic increase]

(*iii*) $P_n(u) \leq f(u)$ *for all* $u \in [a, b]$ *and all* $n = 0, 1, \ldots$

[boundedness from above]

(*iv*) $P_n(u_i) = f(u_i)$ *for all* $i = 0, 1, \ldots, n$ [agreement at nodes]

Konvergencija metode slomljenih pravaca

Theorem 1

Let $f \in Lip_L[a,b]$, $u^\star \in \operatorname{argmin}_{u\in[a,b]} f(u)$, *and let the sequences* (u_n) *and* (P_n) *of numbers and functions respectively, be defined as described in Pijavskij's algorithm. Then*

(i) $\lim\limits_{n\to\infty} P_n(u_{n+1}) = \min\limits_{u\in[a,b]} f(u) = f(u^\star) =: f^\star$;

(ii) *If* $\hat{u}$ *is any accumulation point of the sequence* (u_n) *then* $\hat{u} \in \operatorname*{argmin}\limits_{u\in[a,b]} f(u)$;

(iii) *If* $u^\star = \operatorname*{argmin}\limits_{u\in[a,b]} f(u)$, *i.e.* $u^\star$ *is the only point of the global minimum, then the sequence* (u_n) *converges to* $u^\star$.

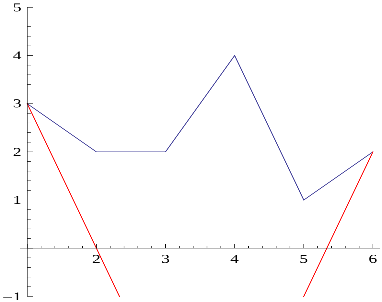
Shubert's algoritam (B. O. Shubert, *An algorithm for searching for a global minimum of a function*, SIAM Journal on Numerical Analysis, **9**(1972), 888–896)

Algorithm (Shubert's algorithm)

Input: $f : [a, b] \to \mathbb{R}, \quad L > 0, \quad n \geq 1$
  {Input the function and the number of iterations;}
  $k = 1$;
  Set $u_1 = U(a, b)$;
  Form a list of intervals $I = \{(a, u_1), (u_1, b)\}$;
  while $k = k + 1; k < n$, **do**
    Remove from I the interval (l, r) with the smallest B-value;
    Set $u_k = U(l, r)$;
    Append (l, u_k) and (u_k, r) to I;
  end while
Output: $\{u_{k+1}, f(u_{k+1})\}$

Shubert algoritam

- Odrediti $(U(a,b),B)$ kao presjek pravaca $x \mapsto f(a) - L(x-a)$ i $x \mapsto f(b) + L(x-b)$ i definirati $u_1 := U(a,b)$;
- definirati $I = \{[a,u_1],[u_1,b]\}$ i njihove B-vrijednosti;
- Ako je manja B-vrijednost na $[a,u_1]$, staviti $u_2 := U(a,u_1)$; Od tri intervala: $[a,u_2]$, $[u_2,u_1]$, $[u_1,b]$ izabrati onaj s najmanjom B-vrijednosti i na njemu ponoviti proceduru.



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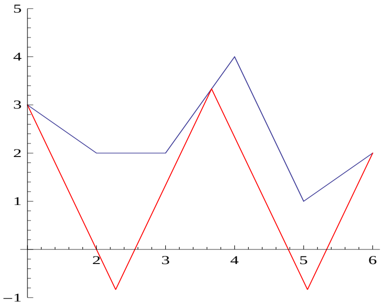

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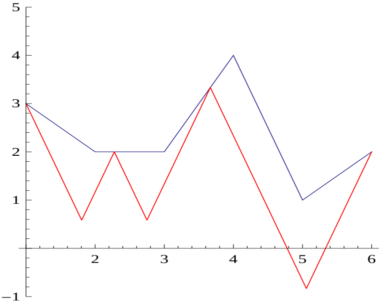

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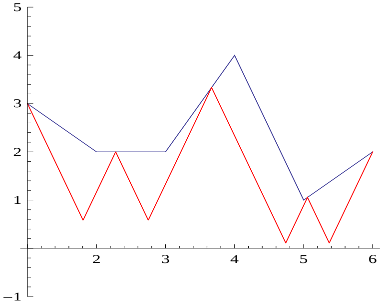

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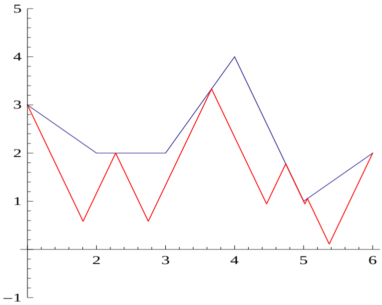

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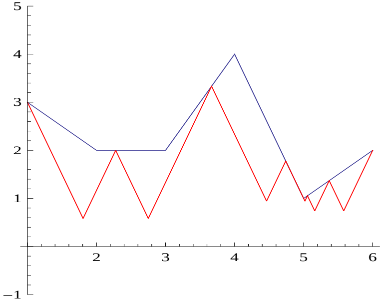

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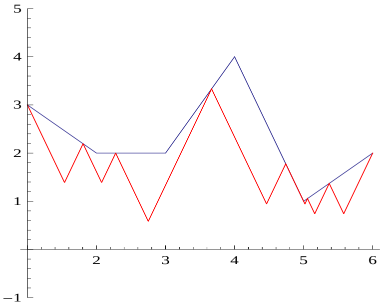

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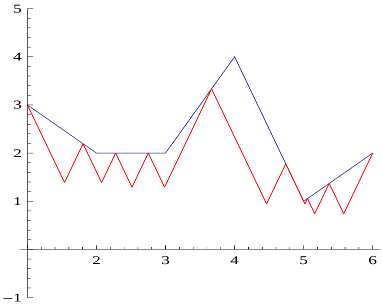
- Ako je manja B-vrijednost na $[a, u_1]$, staviti $u_2 := U(a, u_1)$; Od tri intervala: $[a, u_2]$, $[u_2, u_1]$, $[u_1, b]$ izabrati onaj s najmanjom B-vrijednosti i na njemu ponoviti proceduru.



Shubert algoritam

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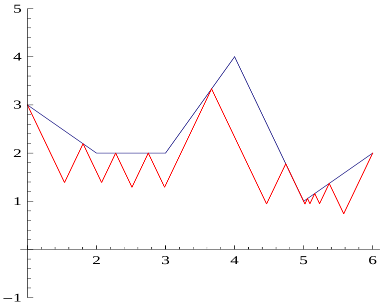

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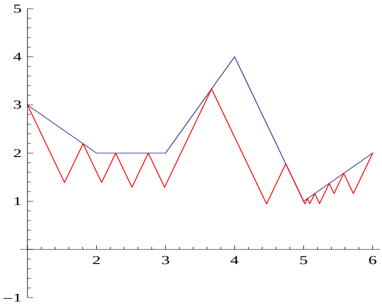

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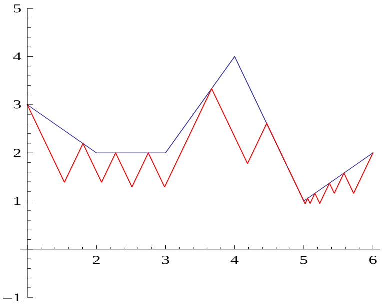

DIRECT algoritam (D. R. Jones, C. D. Perttunen, B. E. Stuckman, *Lipschitzian optimization without the Lipschitz constant*, JOTA **79**(1993), 157–181)

Algorithm (DIRECT algorithm)

Input: $f : [a, b] \to \mathbb{R}, \quad L > 0, \quad n \geq 1$

{Input the function and the number of iterations;}

$k = 1$;

Set $c = \frac{a+b}{2}$, $d = \frac{b-a}{2}$ and $min = \{c, f(c)\}$;

Form a list of intervals $I = \left\{(a, c - \frac{d}{3}), (c - \frac{d}{3}, c + \frac{d}{3}), (c + \frac{d}{3}, b)\right\}$;

Update *min* with the function values at the two new centers $c - 2\frac{d}{3}$ and $c + 2\frac{d}{3}$;

while $k = k + 1$; $k < n$, **do**

    Remove from I the interval (l, r) with the smallest B-value;

    Divide the interval $[a, b]$ into 3 subintervals and append them to I;

    Update *min* with the function values at the centers of new subintervals;

end while

Output: $\{min, f(min)\}$

DIRECT algoritam (D. R. Jones, C. D. Perttunen, B. E. Stuckman, *Lipschitzian optimization without the Lipschitz constant*, JOTA **79**(1993), 157–181)

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Korigirati *min* s vrijednostima u novim centrima $c - 2\frac{d}{3}$ i $c + 2\frac{d}{3}$; Staviti $n := 3$;

K1: Izračunati $\mathcal{B}(c_i) = f(c_i) - Ld$ i odrediti $j = \operatorname*{argmin}_{i=1,\ldots,n} \mathcal{B}(c_i)$;

K2: Interval s centrom u c_j podijeliti na 3 podintervala jednake širine $d := \frac{d}{3}$;
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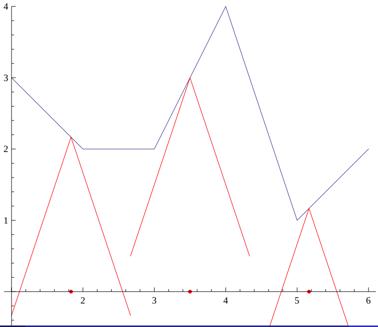

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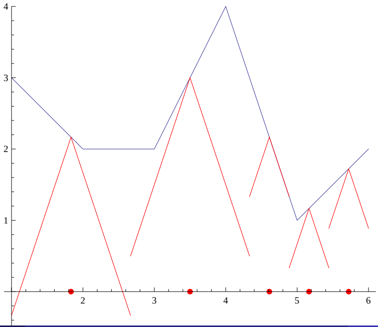

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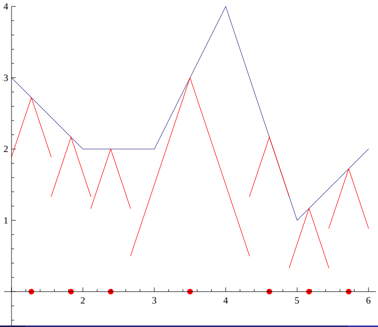

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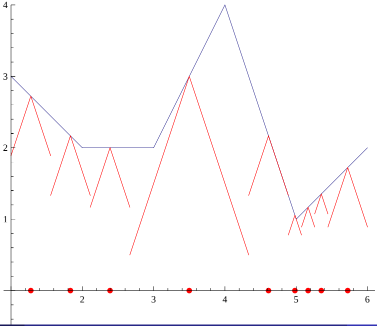

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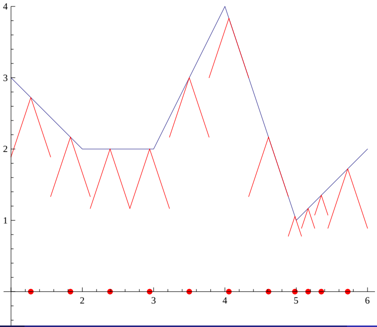

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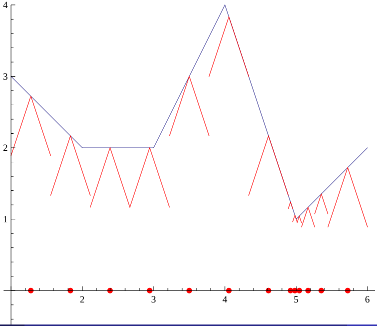

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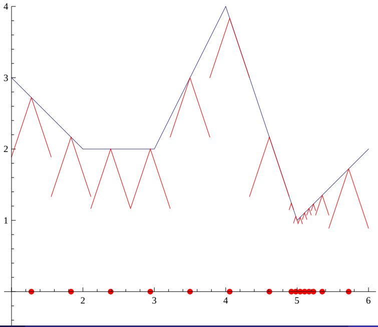

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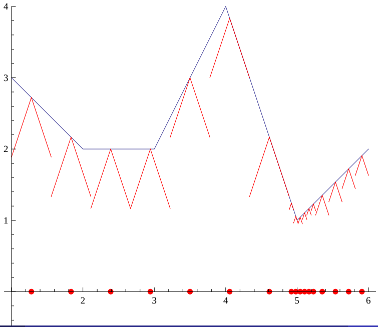

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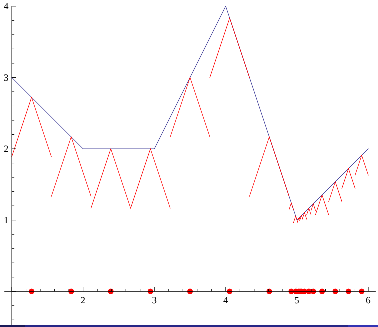

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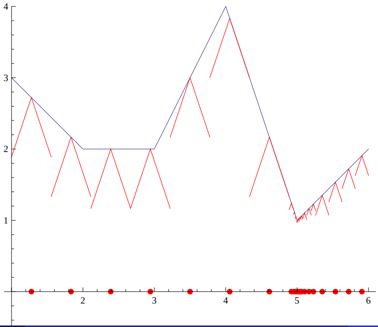

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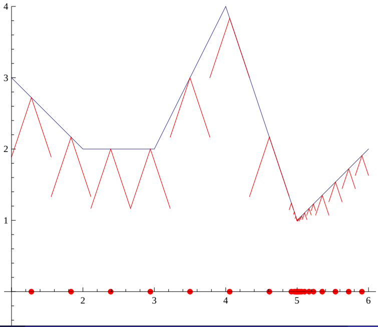
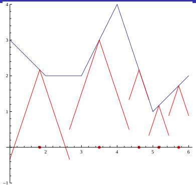
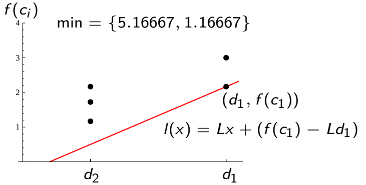


Traženje trenutno optimalnog intervala $\left(d_1 = \frac{b-a}{6},\ d_2 = \frac{b-a}{18},\ d_3 = \frac{b-a}{54},\ d_4 = \frac{b-a}{162}\right)$

2. iteracija





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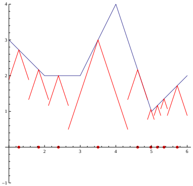
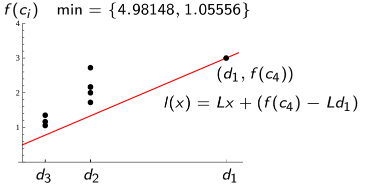
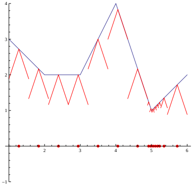
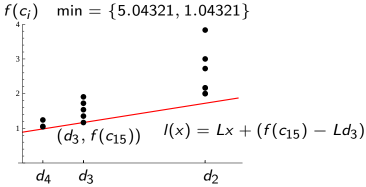




7. iteracija





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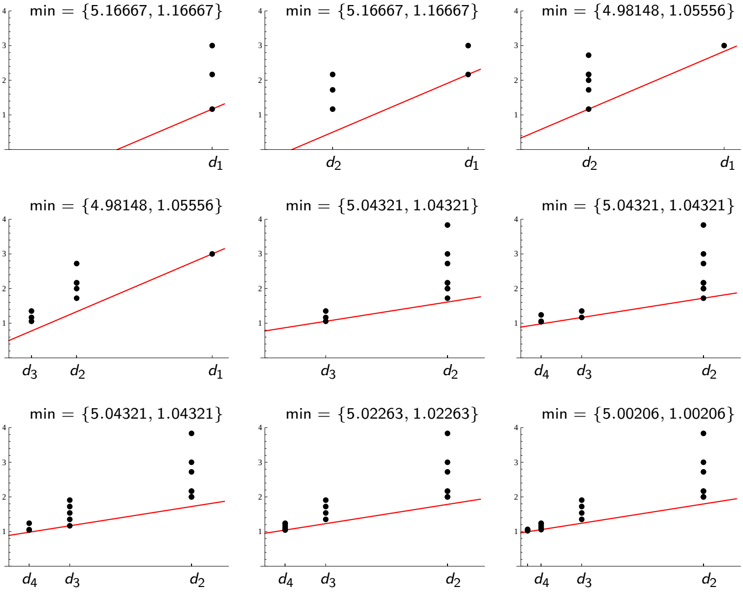
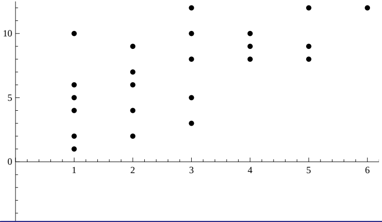


Međutim, kao što se može vidjeti ima i boljih pravaca s manjim koeficijentom smjera (većom B-vrijednosti),a koji imaju spomenuto svojstvo:

$f(c_1) - L_1 d_1 \leq f(c_i) - L_1 d_i$ za sve i
$f(c_2) - L_2 d_2 \leq f(c_i) - L_2 d_i$ za sve i
$f(c_3) - L_3 d_3 \leq f(c_i) - L_3 d_i$ za sve i
$L_1 > L_2 > L_3$ [sve dobre točke leže na konveksnoj ljusci]



Definition 1

Suppose $[a_i, b_i]$, $i \in I = \{1, \ldots N\}$, are intervals obtained at a certain stage of the DIRECT algorithm. The interval $[a_j, b_j]$ with center c_j and half-width $d_j = \frac{b_j - a_j}{2}$ is said to be *potentially optimal* if there exists an $\hat{L} > 0$ such that

$$f(c_j) - \hat{L} d_j \leq f(c_i) - \hat{L} d_i \text{ for all } i \in I = \{1, \ldots, N\}. \quad (1)$$

Međutim, kao što se može vidjeti ima i boljih pravaca s manjim koeficijentom smjera (većom B-vrijednosti),a koji imaju spomenuto svojstvo:

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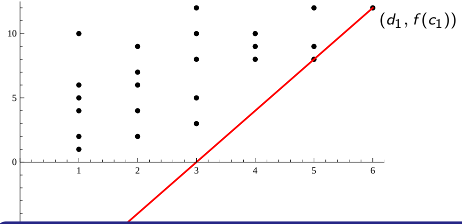
$f(c_3) - L_3 d_3 \leq f(c_i) - L_3 d_i$ za sve i
$L_1 > L_2 > L_3$ [sve dobre točke leže na konveksnoj ljusci]



Definition 1

Suppose $[a_i, b_i]$, $i \in I = \{1, \dots N\}$, are intervals obtained at a certain stage of the DIRECT algorithm. The interval $[a_j, b_j]$ with center c_j and half-width $d_j = \frac{b_j - a_j}{2}$ is said to be *potentially optimal* if there exists an $\hat{L} > 0$ such that

$$f(c_j) - \hat{L} d_j \leq f(c_i) - \hat{L} d_i \text{ for all } i \in I = \{1, \dots, N\}. \quad (1)$$

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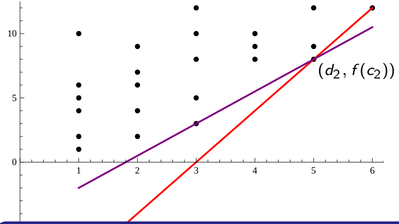

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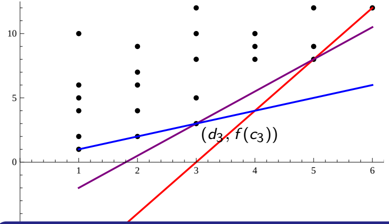

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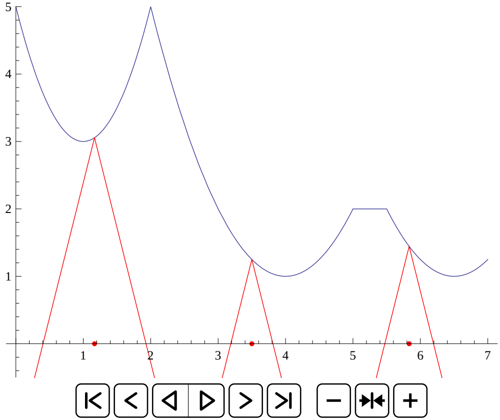
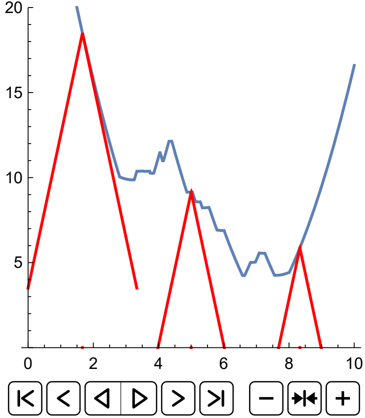
Zašto nije dovoljno dobro izabrati točke s najmanjom vrijednosti funkcije ?

$$f:[1,7]\to\mathbb{R},\qquad f(x)=\begin{cases}2(x-1)^2+3, & 0\le x\le 2,\\ (x-4)^2+1, & 2\le x\le 5,\\ 2, & 5\le x\le 5.5,\\ (x-6.5)^2+1, & 5.5\le x\le 7\end{cases}$$



- $m = 30$; $a = 0$; $b = 10.$; `y=RandomReal[{a,b}, m];`
- $f : [0, 10] \to [0, 20]$, $f(x) = \operatorname*{med}_{i=1,\ldots,m} (y_i - x)^2$;
- $L = 9$;



Lema (Gablonsky,2001)

Lemma 2

Neka je $f \in Lip_L[a,b]$ *Lipschitz neprekidna funkcija s konstantom* $L > 0$, $\{d_i, c_i \colon i \in I\}$ *poluširine i centri intervala nastali dijeljenjem intervala* $[a,b]$ *u algoritmu* DIRECT, $f_{min} = \min_{i \in I} f(c_i)$ *trenutna vrijednost minimuma.*

Interval $[a_j, b_j]$, $j \in I$ *je potencijalno optimalan onda i samo onda ako je*

$$f(c_j) \le f(c_i), \quad \forall i \in I_0, \tag{2}$$

$$\max_{k \in I_L} \frac{f(c_j) - f(c_k)}{d_j - d_k} \le \min_{k \in I_R} \frac{f(c_j) - f(c_k)}{d_j - d_k}, \tag{3}$$

- $I_0 = \{i \in I \colon d_i = d_j\}$ – *indeksi točaka s apscisom* d_j;
- $I_L = \{i \in I \colon d_i < d_j\}$ – *indeksi točaka koje se na grafikonu nalaze lijevo od točaka s apscisom* d_j;
- $I_R = \{i \in I \colon d_i > d_j\}$ – *indeksi točaka koje se na grafikonu nalaze desno od točaka s apscisom* d_j.

- za $i \in I_L$ $(d_j - d_i > 0)$ uvjete (1) možemo zapisati kao

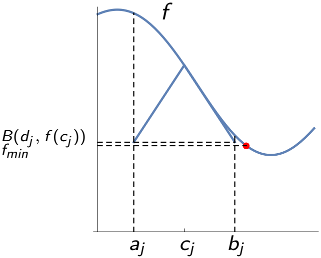
$$\frac{f(c_j) - f(c_i)}{d_j - d_i} \le \max_{k \in I_L} \frac{f(c_j) - f(c_k)}{d_j - d_k} \le \hat{L}, \quad \forall i \in I_L. \tag{4}$$

- za $i \in I_R$ $(d_j - d_i < 0)$ uvjete (1) možemo zapisati kao

$$\frac{f(c_j) - f(c_i)}{d_j - d_i} \ge \min_{k \in I_R} \frac{f(c_j) - f(c_k)}{d_j - d_k} \ge \hat{L}, \quad \forall i \in I_R. \tag{5}$$

Ako je $[a_j, b_j]$ potencijalno optimalan intrval sukladno, onda vrijedi

$$\mathcal{B}(d_j, f(c_j)) = f(c_j) - \hat{L}d_j \leq f(c_j) < f_{min}.$$



Slika: Interval koji nije potencijalno optimalan

$f_{min} \neq 0$:

$$\frac{f_{min} - (f(c_j) - \hat{L}d_j)}{|f_{min}|} > 0 \quad \Longrightarrow \quad \text{postoji } \epsilon > 0, \quad \frac{f_{min} - (f(c_j) - \hat{L}d_j)}{|f_{min}|} \geq \epsilon > 0.$$

Lema (Gablonsky,2001)

Lemma 3

Neka je $f \in Lip_L[a,b]$ *Lipschitz neprekidna funkcija s konstantom* $L > 0$, $\epsilon > 0$, $\{d_i, c_i \colon i \in I = \{1, \ldots, n\}$, *poluširine i centri intervala nastali dijeljenjem intervala* $[a,b]$ *u algoritmu* `DIRECT` *i* $f_{min} = \min\limits_{i \in I} f(c_i)$.
Neka je $S \subset I$ *skup indeksa potencijalno optimalnih intervala i neka je* $\epsilon > 0$.
$\mathcal{B}$*-vrijednost potencijalno optimalnog intervala* $[a_j, b_j]$, $j \in S$ *razlikuje se od aktualnog minimuma* f_{min} *za više od* ϵ *onda i samo onda ako vrijedi*

$$\epsilon \leq \frac{f_{min} - f(c_j)}{|f_{min}|} + \frac{d_j}{|f_{min}|} \min_{k \in I_R} \frac{f(c_j) - f(c_k)}{d_j - d_k}, \qquad \text{ako je } f_{min} \neq 0, \tag{6}$$

odnosno

$$f(c_j) \leq d_j \min_{k \in I_R} \frac{f(c_j) - f(c_k)}{d_j - d_k}, \qquad \text{ako je } f_{min} = 0, \tag{7}$$

gdje je $I_R = \{i \in I \colon d_i > d_j\}$.

Lema (Gablonsky,2001)

Theorem 2

Neka je $f \in Lip_L[a,b]$ *Lipschitz neprekidna funkcija s konstantom* $L > 0$, $\{d_i, c_i \colon i \in I\}$ *poluširine i centri intervala nastali dijeljenjem intervala* $[a,b]$ *u algoritmu* `DIRECT`, $f_{min} = \min\limits_{i \in I} f(c_i)$ *aktualna vrijednost minimuma i* $\epsilon > 0$.
Interval $[a_j, b_j]$, $j \in I$ *je potencijalno optimalan, a njegova* $\mathcal{B}$*-vrijednost razlikuje se od aktualnog minimuma* f_{min} *za više od* ϵ *onda i samo onda ako vrijedi*

$$f(c_j) \le f(c_i), \quad \forall i \in I_0,$$

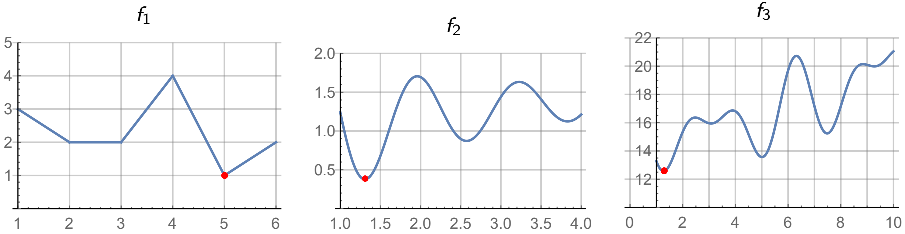
$$\max_{k \in I_L} \frac{f(c_j) - f(c_k)}{d_j - d_k} \le \min_{k \in I_R} \frac{f(c_j) - f(c_k)}{d_j - d_k},$$

$$\begin{cases} \epsilon \le \frac{f_{min} - f(c_j)}{|f_{min}|} + \frac{d_j}{|f_{min}|} \min\limits_{k \in I_R} \frac{f(c_j) - f(c_k)}{d_j - d_k}, & \textit{ako je } f_{min} \neq 0, \\ f(c_j) \le d_j \min\limits_{k \in I_R} \frac{f(c_j) - f(c_k)}{d_j - d_k}, & \textit{ako je } f_{min} = 0, \end{cases}$$

gdje je $I_0 = \{i \in I \colon d_i = d_j\}$, $I_L = \{i \in I \colon d_i < d_j\}$, $I_R = \{i \in I \colon d_i > d_j\}$.

Komparacija metoda

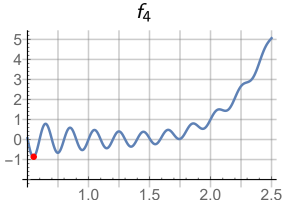
- $f_1 : [1, 6] \to \mathbb{R}, \qquad L = 3, \qquad min = (5, 1);$
- $f_2 : [1, 4] \to \mathbb{R}, \quad f(x) = \frac{\sin(5x-2)}{x} + \frac{x}{10} + 1,$
  $L = 4.99, \qquad min = (1.307, 0.378);$
- $f_3 : [1, 10] \to \mathbb{R}, \quad f(x) = 10 + x - 2\ln\frac{x}{10} + 2\cos(2x) + 1.5\cos(3x),$
  $L = 8.759, \qquad min = (1.273, 12.571);$

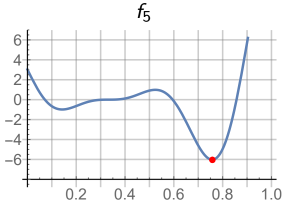


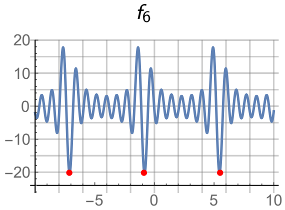
Komparacija metoda

- $f_4 : [0.5, 2.5] \to \mathbb{R}, \quad f(x) = \frac{\sin(10\pi x)}{2x} + (x-1)^4,$
  $L = 31.916, \qquad min = (0.548, -0.869);$
- $f_5 : [0, 1] \to \mathbb{R}, \quad f(x) = (6x-2)^2 \sin(12x-4),$
  $L = 140.849, \qquad min = (0.757, -6.021);$
- $f_6 : [10, 10] \to \mathbb{R}, \quad f(x) = -\sum_{j=1}^{6} j \sin((j+1)x + j), \qquad L = 111.118,$

  $min_1 = (-7.110, fun), \quad min_2 = (0.155, fun), \quad min_3 = (5.457, fun), \quad fun = -20.253.$







Komparacija metoda

The Lipschitz constant of these functions can be easily found as the maximum of the absolute value of the derivative. We compare Pijavskij-Shubert's algorithm and DIRECT algorithm with known Lipschitz constant and DIRECT algorithm without Lipschitz constant.

The Pijavskij-Shubert's algorithm is initialized with left endpoint of the domain.

All algorithms are run until the current approximation x' satisfies the condition

$$\frac{f(x') - f(x^*)}{|f(x^*)|} < 10^{-5},$$

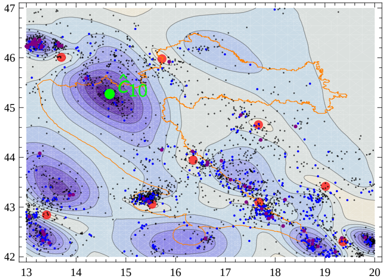
where x^* is the true minimizer.

Method	f_1		f_2		f_3		f_4		f_5		f_6	
	k	N_f	k	N_f	k	N_f	k	N_f	k	N_f	k	N_f
Pijavskij-Shubert	28	28	51	51	31	31	84	84	34	34	54	54
DIRECT with L	21	43	18	37	25	51	40	81	33	67	70	141
DIRECT without L	12	107	5	25	7	37	10	53	6	35	7	29

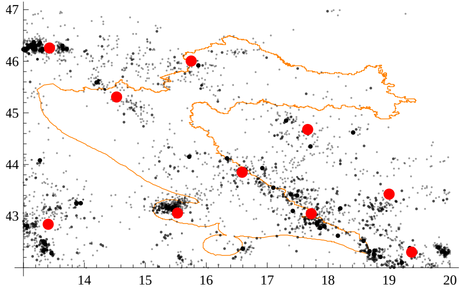
Tablica: The number of iterations k and the number of objective function evaluations N_f for each method and for each test function

Primjer: detekcija centara seizmičke aktivnosti

(a) DIRECT algoritam



(b) k-means algoritam



$$F\colon [13,20]\times[42,47]\to\mathbb{R}_+,$$

$$F(c)=\sum_{i=1}^{m}\min\{d(\hat{c}_1,a_i),\ldots,d(\hat{c}_{k-1},a_i),d(c,a_i)\}$$