Statistical estimation of the object's area from the image contaminated with additive noise

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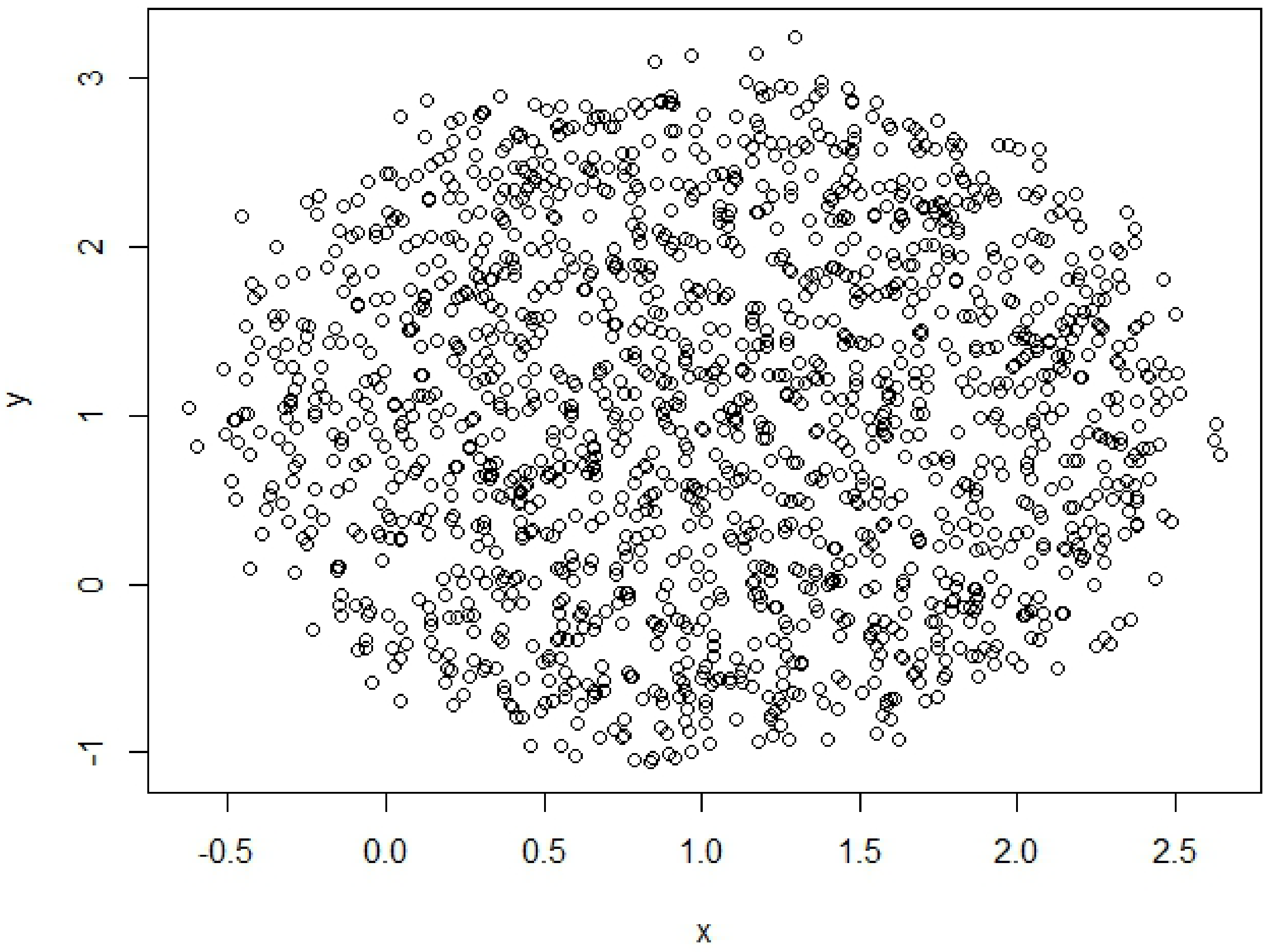
Motivation



Motivation

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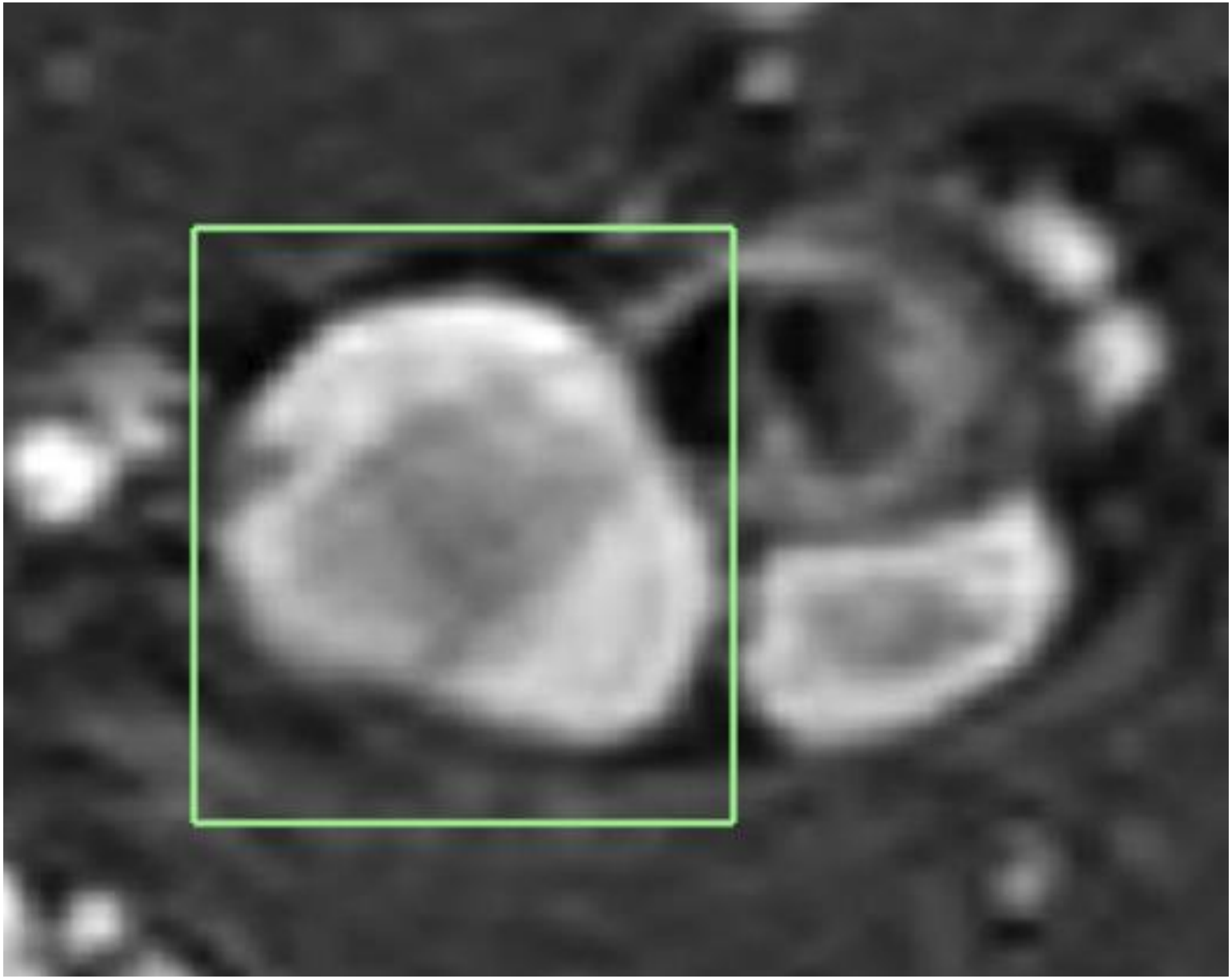


Motivation

Noise on images

- causes:
  - the presence of unwanted structures
  - imperfection of measuring equipment

- various medical images (ultrasound, MRI, X-ray)
- recording microorganisms fluorescent microscope
- reconstruction of objects recorded by GPR (*Ground Penetrating Radar*)

Motivation

Area estimation

- approximation by a circle or ellipse
- existing methods:
  - Hough Transformation [1, 2]
  - *EDCircles* [3]
  - *Fornaciari* [4]

- not designed for detection of an object with unsharp edges

Motivation

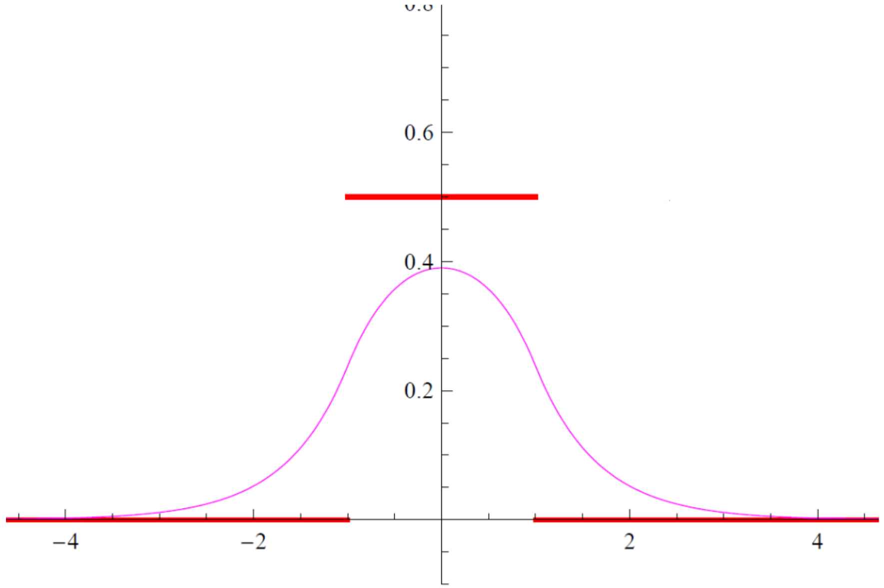
Area estimation

1. P.V.C. Hough, **Methods and means for recognizing complex patterns**, US Patent, 3069654, 1962.
2. E.R. Davies, **A modified Hough scheme for general circle location**, *Pattern Recognition Letters*, vol. 7, pp. 37–43, 1988.
3. C. Akinlar, C. Topal, **EDCircles: A real-time circle detector with a false detection control**, *Pattern Recognition*, vol. 46 (3), pp. 725–740, 2013.
4. M. Fornaciari, A. Prati, R. Cucchiara: **A fast and effective ellipse detector for embedded vision applications**, *Pattern Recognition*, vol. 47 (11), pp. 3693-3708, 2017.

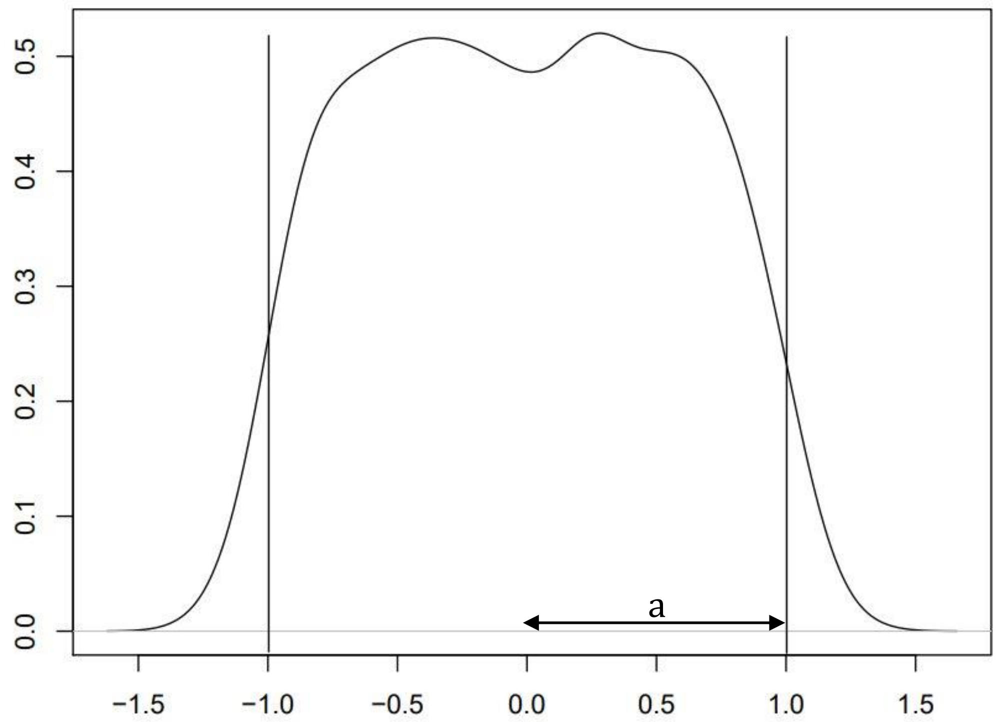
Statistical model

$$X = U + \varepsilon$$

- U and ε are independent random variables
- U – uniform
- ε – error (zero mean, symmetric density)



Statistical model



Goals:

1. Estimate $a > 0$ (half-width of a uniform distribution)

2. Estimate σ^2 (error variance)

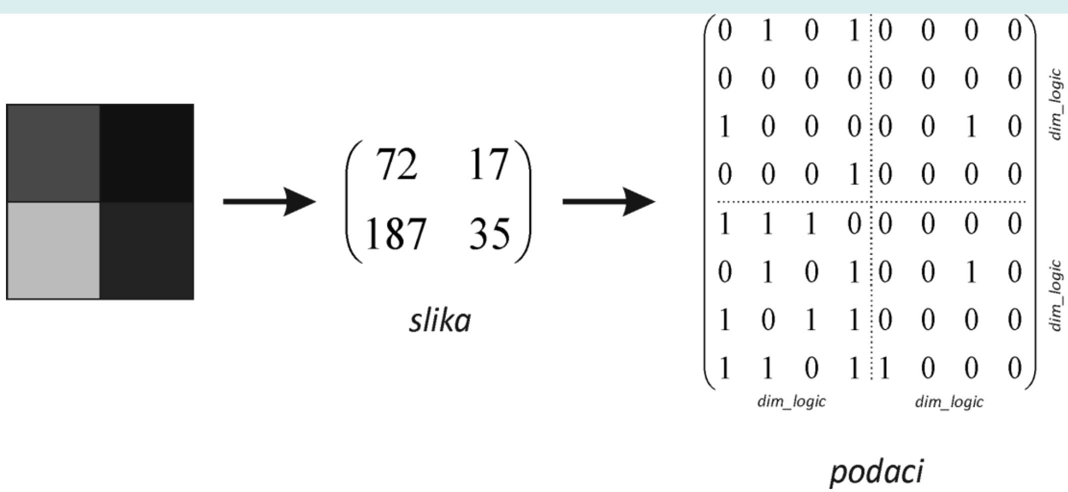
Statistical model

5. M. Benšić, K. Sabo, **Estimating the width of a uniform distribution when data are measured with additive normal errors with known variance**, *Computational Statistics and Data Analysis*, vol. 51, pp. 4731–4741, 2007.
6. M. Benšić, K. Sabo, **Border estimation of a two-dimensional uniform distribution if data are measured with additive error**, *Statistics – A Journal of Theoretical and Applied Statistics*, vol. 41, pp. 311–319, 2007.
7. M. Benšić, K. Sabo, **Estimating a uniform distribution when data are measured with a normal additive error with unknown variance**, *Statistics - A Journal of Theoretical and Applied Statistics*, vol. 44, pp. 235–246, 2010.
8. M. Benšić, K. Sabo, **Uniform distribution width estimation from data observed with Laplace additive error**, *Journal of the Korean Statistical Society*, vol. 45, pp. 505–517, 2016.

Application of the model to edge detection

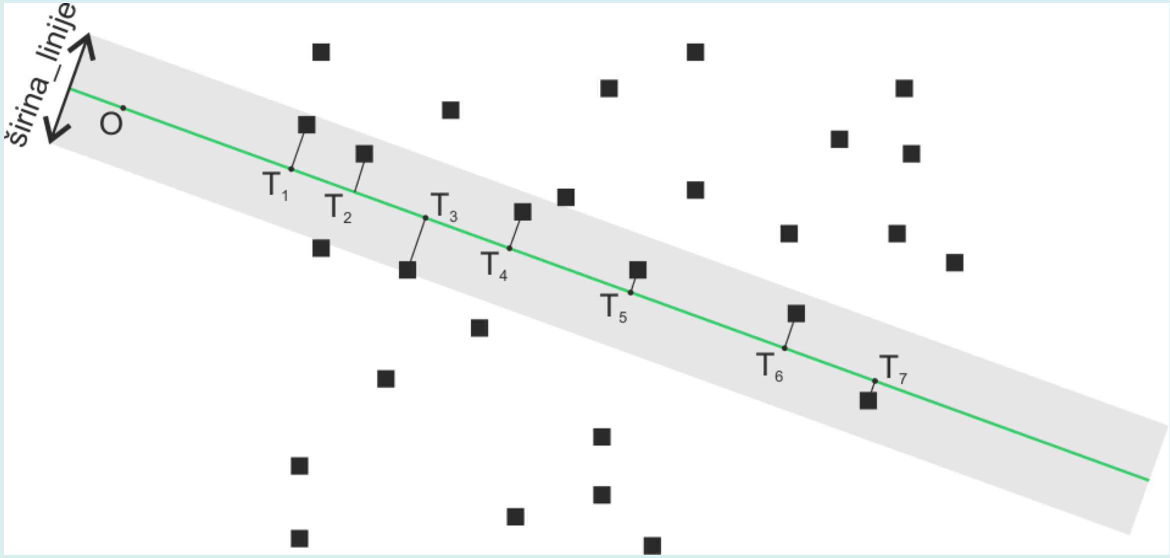
ALGORITHM 1:

1. Represent each pixel of the image by a matrix of $dim\_logic \times dim\_logic$ boolean variables
2. Uniformly distribute n TRUE values in each of these matrices, $n \sim$ brightness of the observed pixel
3. in the described way transform the whole image into a matrix of logical data values



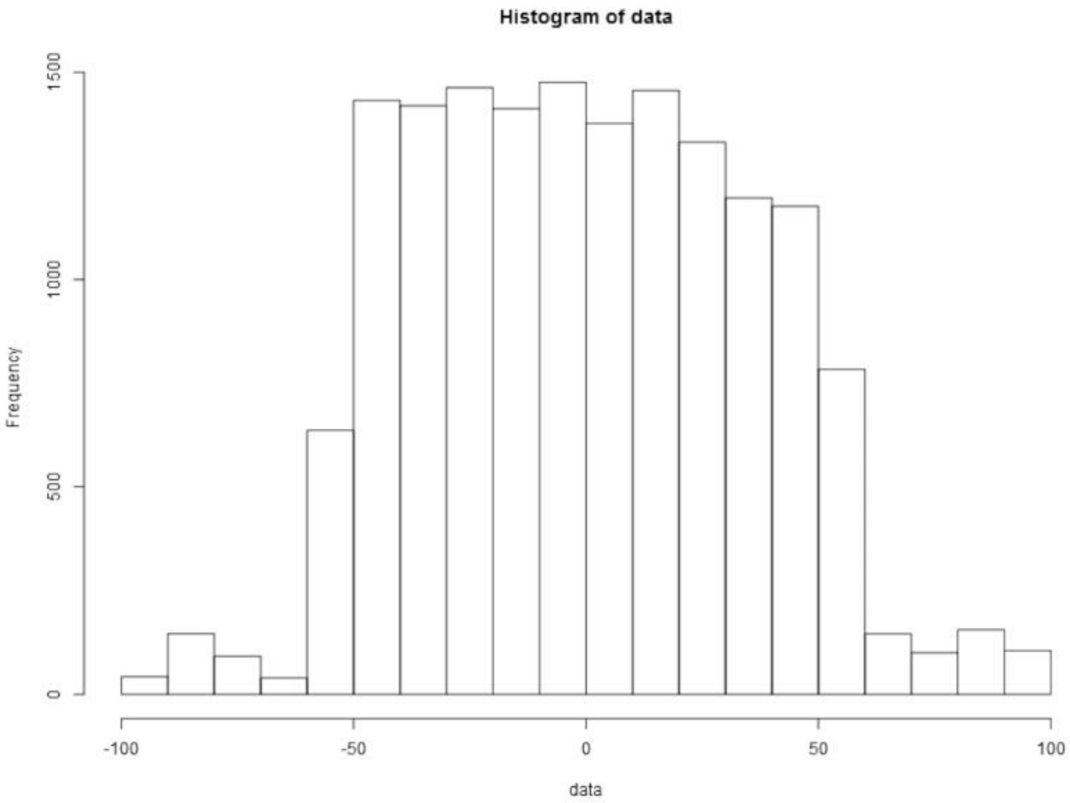
Application of the model to edge detection

4. Orthogonally map each point whose distance from the intersection line is $< w$ on the intersection line



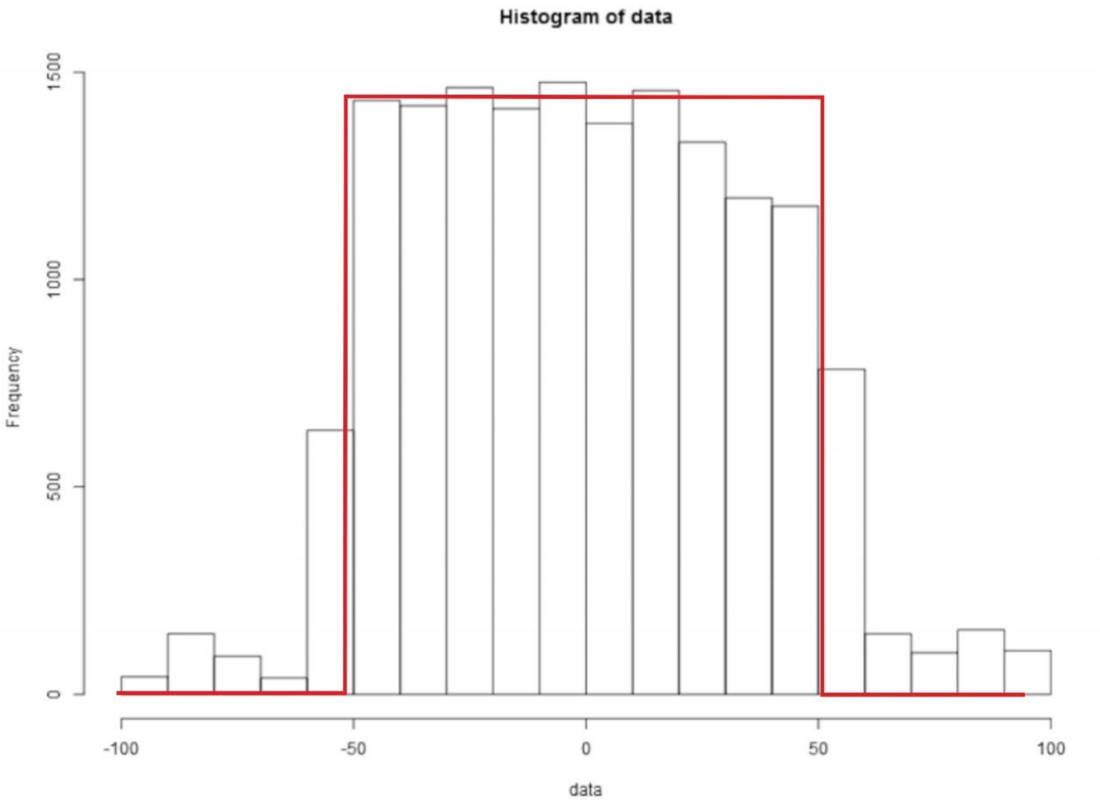
5. calculate the **distance** of all orthogonally mapped points from an arbitrary fixed point on the intersection line

Application of the model to edge detection



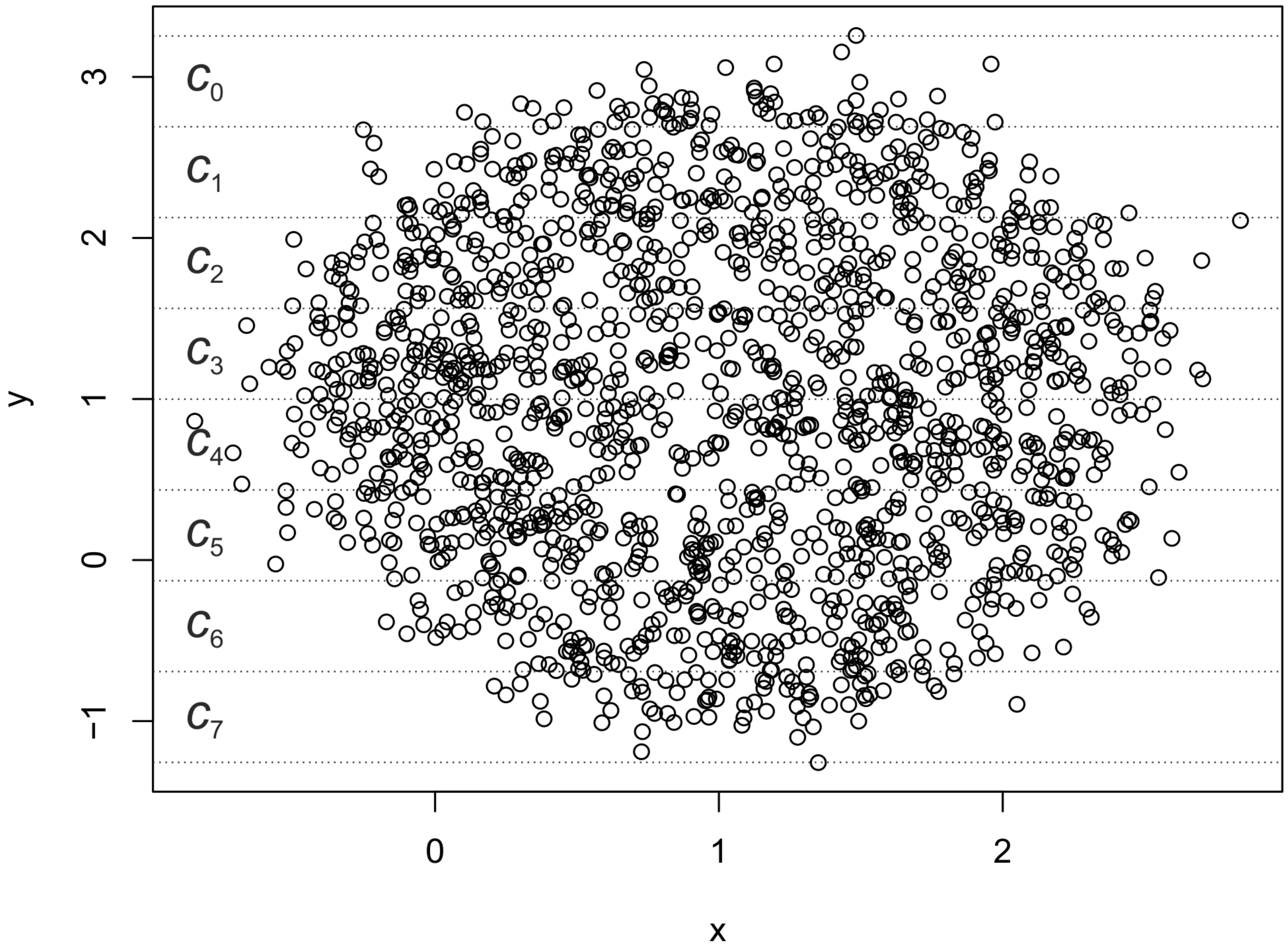
Application of the model to edge detection

- half-width of a uniform distribution:



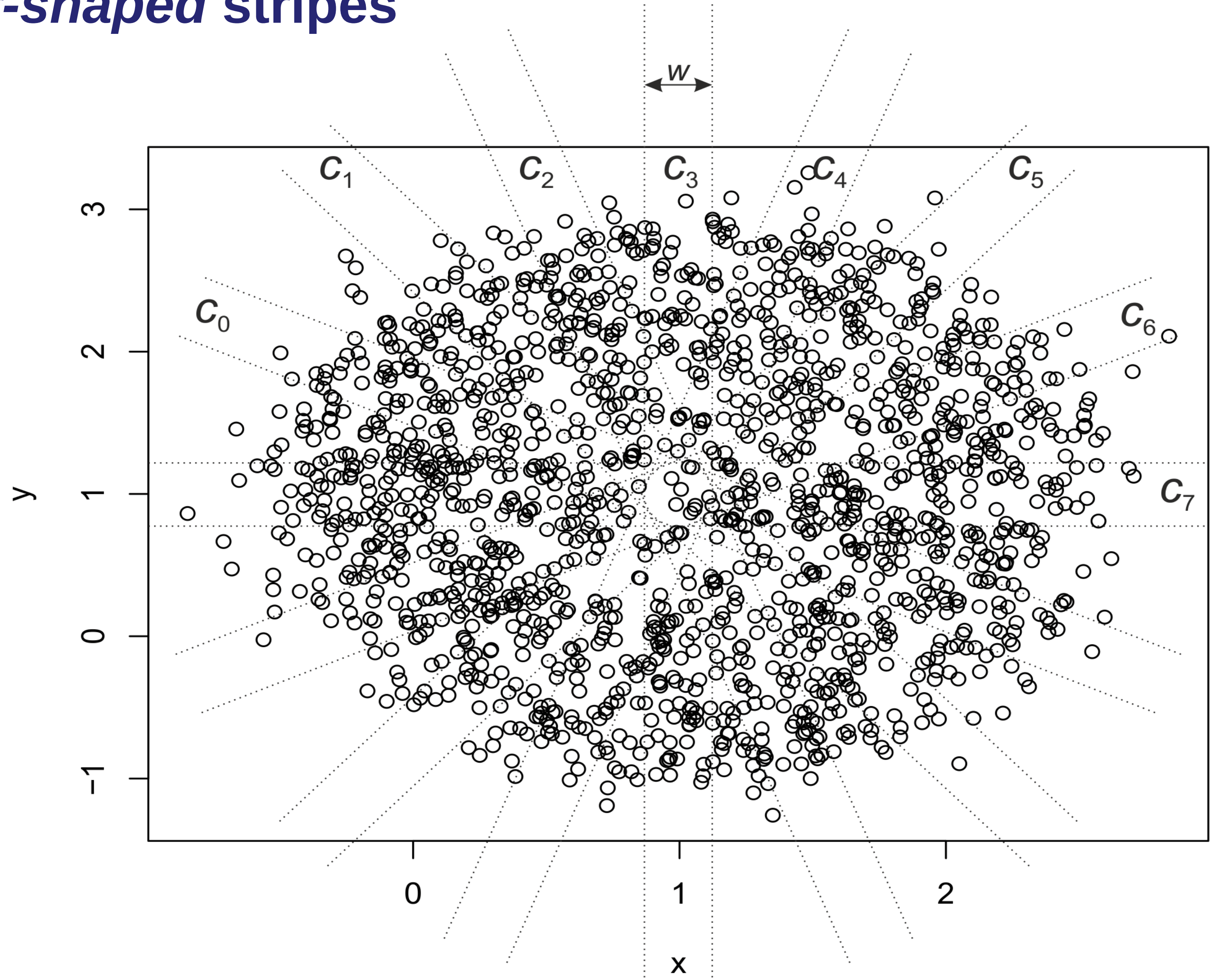
Application of the model to area detection

Horizontal and vertical stripes



Application of the model to area detection

Star-shaped stripes

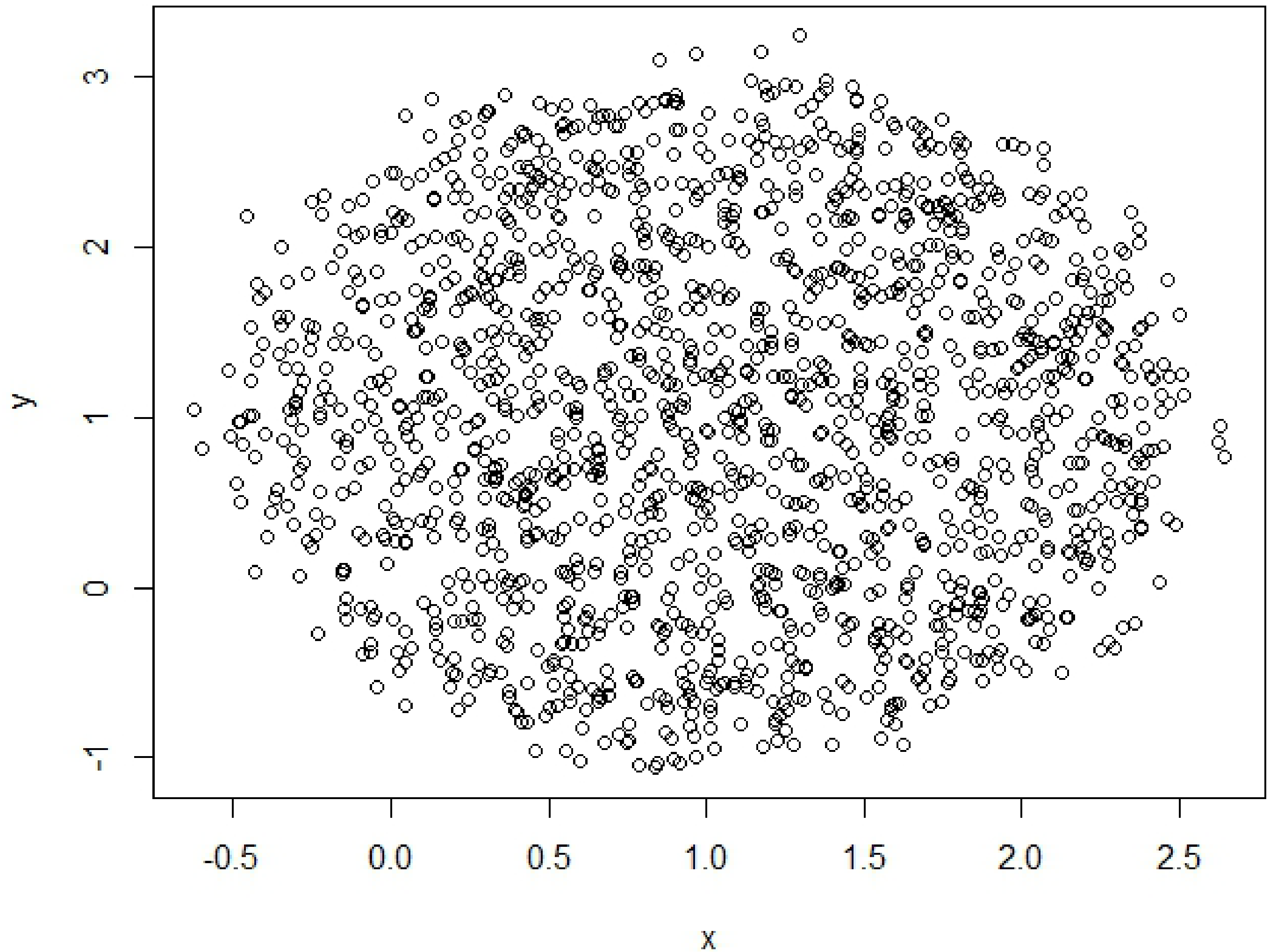


Application of the model to area detection

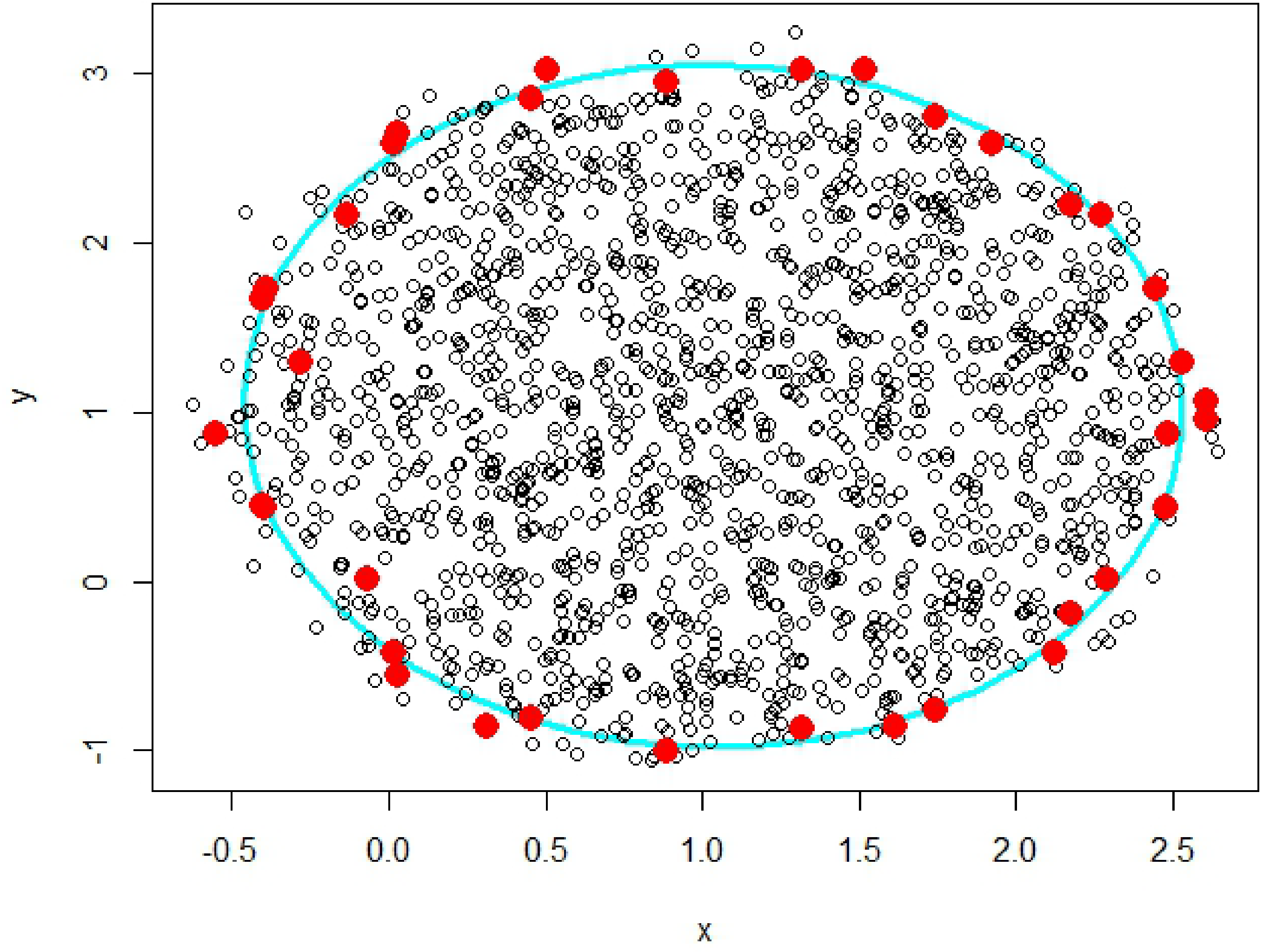
- *least squares circle fit* algorithm [9]
- *least squares ellipse fit* algorithm [10]

9. I. Kåsa, **A circle fitting procedure and its error analysis**, *IEEE Transactions on instrumentation and measurement*, vol. 1001(1), pp. 8-14, 1976.
10. A. Fitzgibbon, M. Pilu, R.B. Fisher, **Direct least square fitting of ellipses**, *IEEE Transactions on pattern analysis and machine intelligence*, vol. 21(5), pp. 476–480, 1999.

Application of the model to area detection



Application of the model to area detection



Implementation

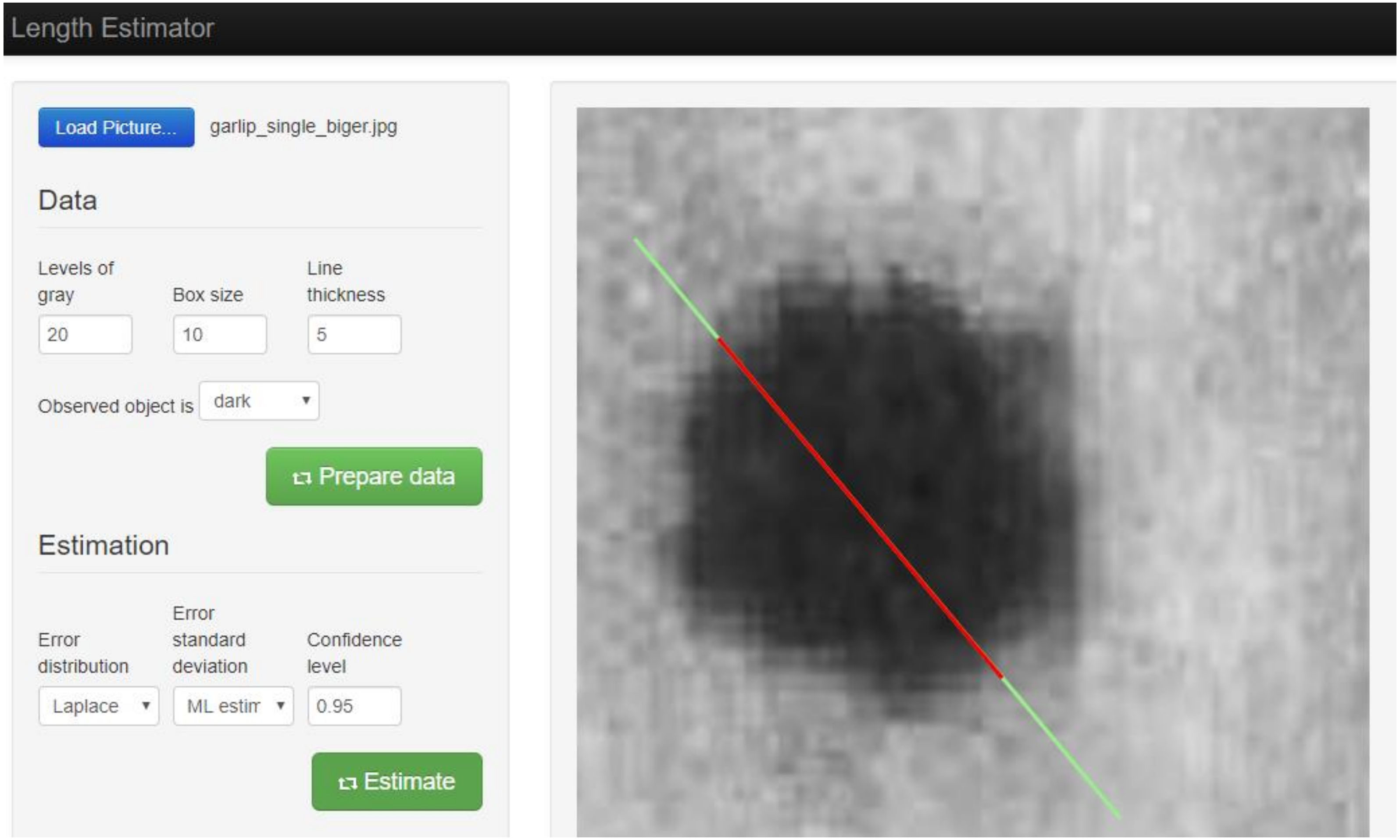
- package *LeArEst* for *R* programming language
  https://cran.r-project.org/package=LeArEst

- package modules:

  1. function for estimating the width of uniform distribution for data contaminated with additive error
  2. function for estimating the area of a circular or ellipsoidal object given numerically by a set of two-dimensional points
  3. web application for length estimation of intersection of the object with an arbitrary line
  4. web application for area estimation of the circular or ellipsoidal object

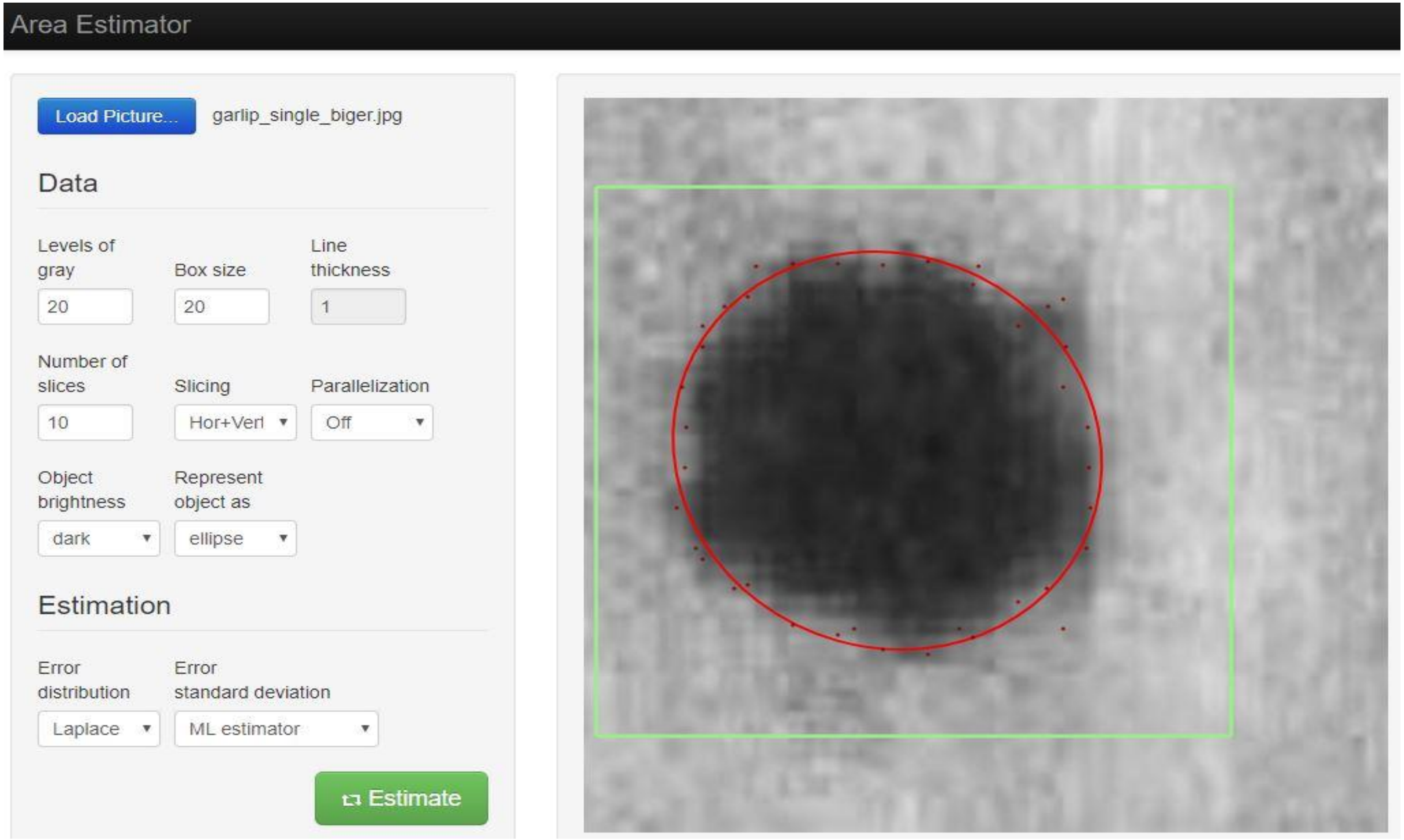
Implementation

Length estimation of intersection of the object with an arbitrary line



Implementation

Area estimation of the circular or ellipsoidal object



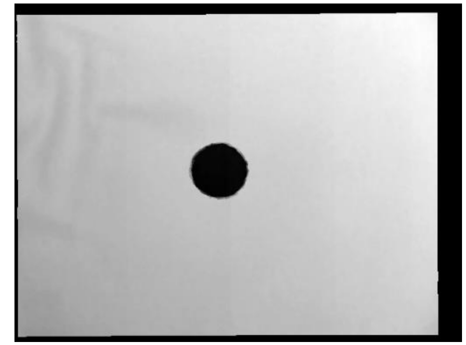
Comparison with existing methods

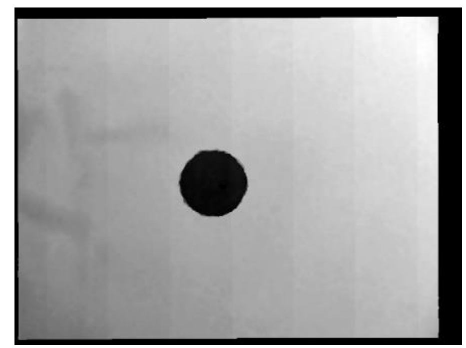
- various objects have been recorded by RGB-D camera which records exact distances of the individual pixels of the captured image from the camera lens

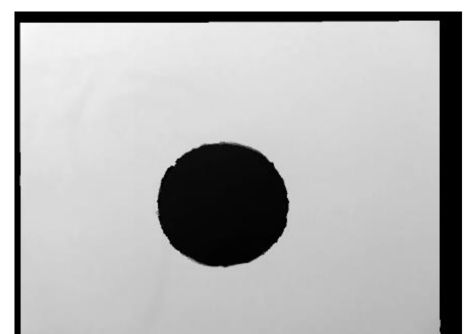
- result: grey scale image – brightness of each pixel of the image is proportional to the distance of the corresponding point from the camera

Comparison with existing methods



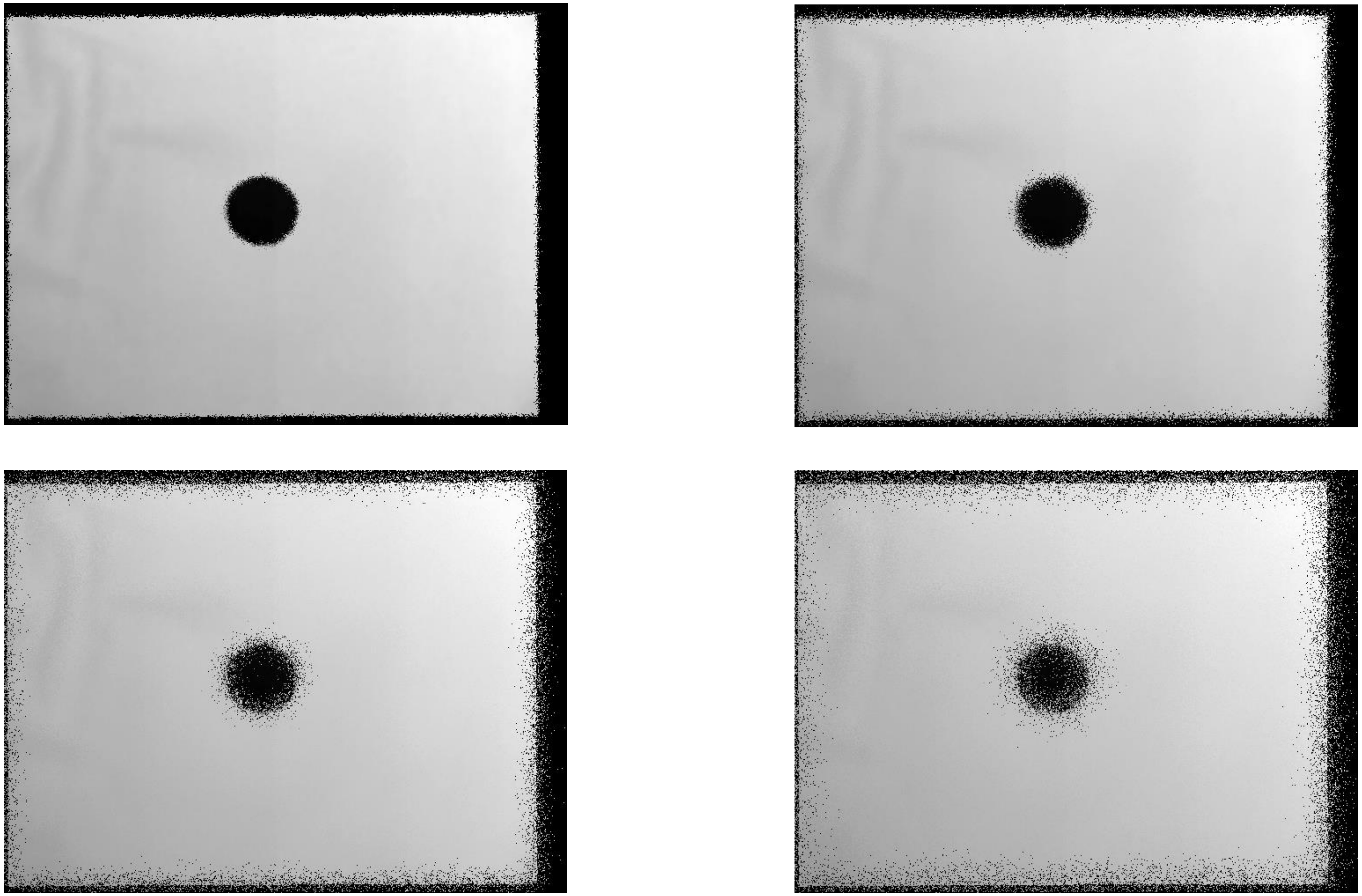


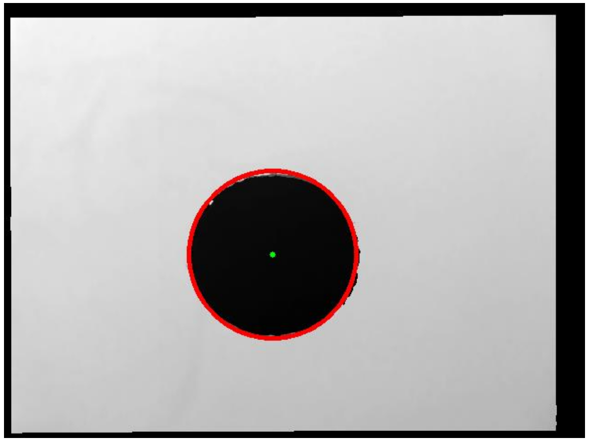
Comparison with existing methods

- normal additive error (noise) of various standard deviation (2, 5, 10 and 15 pixel widths) is added to depth images



Comparison with existing methods

- area detection using:
  - LeArEst
  - Hough Transformation
  - EDCircles
  - Fornaciari



=> estimated area of the object

Comparison with existing methods

- *tin* object (d = 0.103 m, true area = $8.33 \cdot 10^{-3}$ m^2)

σ / d [%]	**LeArEst [10^{-3} m^2]**		**HT [10^{-3} m^2]**		**EDCircles [10^{-3} m^2]**	
	SE	bias	SE	bias	SE	bias
2.47	0.156	0.568	1.15	0.761	1.38	1.46
6.17	0.102	0.513	1.30	0.500	0.143	0.841
12.3	0.123	0.211	2.23	0.787	0.143	0.644
18.9	0.296	0.018	1.91	-0.613	0.198[a]	0.730 [a]

a. Method did not detect an object in three images
b. Fornaciari method did not detect the object in any image set

Comparison with existing methods

- *CD-spindle* object (d = 0.139 m, true area = $15.15 \cdot 10^{-3}$ m^2)

σ / d [%]	**LeArEst [10^{-3} m^2]**		**HT [10^{-3} m^2]**		**EDCircles [10^{-3} m^2]**	
	SE	bias	SE	bias	SE	bias
1.96	0.293	1.72	3.16	0.516	0.208	1.57
4.90	0.198	1.80	3.39	1.63	0.237	1.44
9.80	0.237	1.92	2.87	-1.13	0.262	1.48
14.7	0.567	1.59	2.59	-1.83	0.207	1.48

Comparison with existing methods

- *trashcan* object ($d = 0.240$ m, true area $= 45.23 \cdot 10^{-3}$ m^2)

σ / d [%]	**LeArEst [10^{-3} m^2]**		**HT [10^{-3} m^2]**		**EDCircles [10^{-3} m^2]**	
	SE	bias	SE	bias	SE	bias
1.05	0.410	3.33	5.36	4.03	0.303	3.00
2.64	0.385	3.17	8.53	1.90	0.405	3.08
5.29	0.455	3.27	9.96	4.11	0.508	3.32
7.93	2.61	1.36	13.4	-4.34	1.23[a]	3.06 [a]

a. Method did not detect an object in one image

Real-world application

- Differences between estimated and true areas for all estimation methods for *clean* images

object	difference [10^{-3} m^2]			
	LeArEst	HT	EDCircles	Fornaciari
tin	0.229	1.86	0.681	N/A [a]
CD-spindle	1.34	4.67	1.30	3.99
trashcan	3.28	3.67	3.66	12.1

a. Method did not detect an object

Conclusion

- *LeArEst* method generally performs very well when dealing with clear images, as well as with images with an additive error
- noisy images: biases are similar for *HT* and *LeArEst*, but the *LeArEst* has much smaller variations in estimated areas (standard errors)
- *HT* occasionally produced outliers in estimated areas
- *EDCircles* gave estimation results comparable to *LeArEst*, but sometimes fails to detect an object on image contaminated with much additive noise
- *Fornaciari* method is not comparable to the other three observed methods because it fails to detect an object if any additive error is present on the image

Future work

- estimating the area of objects that are not necessarily circular or ellipsoidal (represent the object as convex polygon)

- implementing some asymmetric error distributions – useful for the cases when we have different amount of noise in the tails

Questions?