3.4 Eksponencijalne i logaritamske nejednadžbe

Napomena.

$$a > 1 \Longrightarrow (a^x \leq a^y \Leftrightarrow x \leq y)$$
$$a \in (0,1) \Longrightarrow (a^x \leq a^y \Leftrightarrow x \geq y)$$

Zadatak 7. Riješite sljedeće nejednadžbe:

a) $11^{\frac{1}{x}} < \left(\frac{1}{11}\right)^{\frac{1}{3-x}}$

b) $0.8 \cdot \left(\frac{4}{5}\right)^{\frac{1}{x-1}} < \left(\frac{5}{4}\right)^{x+\frac{1}{2}}$
Rješenje: $x \in \langle -1, \frac{1}{2}\rangle \cup \langle 1, \infty\rangle$

c) $3^{\frac{1}{x}} + 3^{3+\frac{1}{x}} > 84$
Rješenje: $x \in \langle 0, 1\rangle$

d) $2^{2x+5} - 3 \cdot 2^{x+2} + 1 \leq 0$

e) $3 \cdot 2^{1-x} \geq 1 + 2^x$

f) $0.4^x - 2.5^{x+1} \geq 1.5$
Rješenje: $x \in \langle -\infty, -1]$

g) $6^{2x+3} < 2^{x+7} \cdot 3^{3x-1}$

Zadatak 8. Riješite sljedeće nejednadžbe:

a) $\dfrac{9^x - 3^{x+1} + 2}{\sqrt{2-x}} \geq 0$

b) $\dfrac{8^x - 4^x - 2^{x+1}}{\sqrt{9-x^2}} \leq 0$
Rješenje: $x \in \langle -3, 1]$

c) $\dfrac{3^{2x+1} - 4 \cdot 3^x + 1}{3^x - 9^x} \leq 0$

d) $\dfrac{2^{2x+3} - 3 \cdot 2^{x+1} + 1}{2^{1-x} - 1} > 0$
Rješenje: $x \in \langle -\infty, -2\rangle \cup \langle -1, 1\rangle$

e) $\dfrac{3^x}{3^x-1} - \dfrac{1}{3^x+1} \leq 0$

f) $\dfrac{2^x}{5^{x-1}} + 3 < \dfrac{5^x}{2^{x-1}}$
Rješenje: $x \in \langle 1, \infty \rangle$

g) $\dfrac{1}{2^x-4} > \dfrac{1}{2^x-1}$
Rješenje: $x \in \langle -\infty, 0\rangle \cup \langle 2, \infty \rangle$

Napomena.

$$a > 1 \Longrightarrow (\log_a x \leq \log_a y \Leftrightarrow x \leq y)$$
$$a \in (0,1) \Longrightarrow (\log_a x \leq \log_a y \Leftrightarrow x \geq y)$$

Zadatak 9. Riješite sljedeće nejednadžbe:

a) $\log_3 \dfrac{x-2}{x} < 2$

b) $\log_2 \dfrac{3x-1}{3x+1} > 1$
Rješenje: $x \in \langle -1, -\frac{1}{3} \rangle$

c) $\log_{\frac{1}{3}}(x+1) - \log_3 x > \log_{\frac{1}{3}} 2$
Rješenje: $x \in \langle 0, 1 \rangle$

d) $\log_2 x - \dfrac{2}{\log_2 x - 1} \leq 0$

e) $\dfrac{1}{\log x} - \dfrac{1}{\log x - 1} < 1$
Rješenje: $x \in \langle 0, 1 \rangle \cup \langle 10, \infty \rangle$

Zadatak 10. Riješite sljedeće nejednadžbe:

a) $\log_{1+x}(2-x) \leq 1$

b) $\log_x \dfrac{3}{8-2x} \geq -2$
Rješenje: $x \in \langle 0, 1 \rangle \cup [\frac{4}{3}, 4\rangle$

c) $\log_x 2 \cdot \log_2 4x > 1$

d) $\log\dfrac{10}{x}\cdot\log(10x)\geq\log(\dfrac{1}{10x})$
Rješenje: $x\in[\frac{1}{10},100]$

e) $\log x^2+\log^2 x\leq 3$
Rješenje: $x\in[\frac{1}{10^3},10]$

3.5 Trigonometrijske nejednadžbe

Zadatak 11. Riješite sljedeće nejednadžbe:

a) $|\cos x|<\dfrac{\sqrt{3}}{2}$

b) $\operatorname{tg} x\leq 2$

c) $\sin^2 2x\leq\sin 2x$

d) $\sin^2 x+2\sin x>0$
Rješenje: $x\in\bigcup\limits_{k\in\mathbb{Z}}\langle 2k\pi,\pi+2k\pi\rangle$

e) $2\cos^2 x+\cos x<1$

f) $2\sin^4 x-3\sin^2 x+1\geq 0$

g) $2\cos^2 x+5\cos x+2\geq 0$
Rješenje: $x=\bigcup\limits_{k\in\mathbb{Z}}[\frac{-2\pi}{3}+2k\pi,\frac{2\pi}{3}+2k\pi]$

Zadatak 12. Riješite sljedeće nejednadžbe:

a) $3\sin^2 x+5\cos^2 x-8\sin x\cos x>0$

b) $6\sin^2 x-\cos^2 x-\sin x\cos x>2$
Rješenje: $x=\bigcup\limits_{k\in\mathbb{Z}}\langle\operatorname{arctg}(\frac{3}{4})+k\pi,\frac{3\pi}{4}+k\pi\rangle$

c) $1+\sin x+\cos x<0$

d) $\sin x+\cos x>1$
Rješenje: $x=\bigcup\limits_{k\in\mathbb{Z}}\langle 2k\pi,\frac{\pi}{2}+2k\pi\rangle$

e) $2\sin^2 x \geq \sin 2x$
Rješenje: $x = \bigcup_{k\in\mathbb{Z}} [\frac{\pi}{4} + k\pi, \pi + k\pi\rangle$

f) $\sin x + \cos x + \sin 2x > 1$

g) $\cos x - \sin x + \sin x \cos x > \frac{1}{2}$
Rješenje: $x = \bigcup_{k\in\mathbb{Z}} \langle\frac{-3\pi}{4} + 2k\pi, \frac{\pi}{4} + 2k\pi\rangle$

h) $\cos 8x - \cos 4x \leq 0$
Rješenje: $x = \bigcup_{k\in\mathbb{Z}} [\frac{-\pi}{6} + \frac{k\pi}{3}, \frac{\pi}{6} + \frac{k\pi}{3}]$

i) $\sin x \cdot \cos 5x < \sin 9x \cdot \cos 3x$

j) $\sin x \cdot \sin 3x > \sin 5x \cdot \sin 7x$
Rješenje: $x = \bigcup_{k\in\mathbb{Z}} (\langle\frac{\pi}{8} + \frac{k\pi}{2}, \frac{\pi}{4} + 2k\pi\rangle \cup \langle\frac{-\pi}{4} + \frac{k\pi}{2}, \frac{-\pi}{8} + 2k\pi\rangle)$