Odjel za matematiku, Sveučilište u Osijeku
4. srpnja 2016.

Pismeni ispit iz Integralnog računa

Zadatak 1 Izračunajte $\int_0^1 x^2 \cdot \ln \frac{2-x}{2+x} \mathrm{d}x$.

Zadatak 2 Ispitajte konvergenciju integrala $\int_0^{+\infty} \frac{\mathrm{d}x}{e^{x^4}+e^{-x^4}}$.

Zadatak 3 Ispitajte konvergenciju sljedećeg reda, te ukoliko je konvergentan, izračunajte njegovu sumu

$$\sum_{n=3}^{\infty} \Big(\frac{1}{n^2-2n} + \frac{2 \cdot 3^{n-1}}{7^{n+1}} \Big).$$

Zadatak 4 Odredite područje konvergencije i radijus konvergencije sljedećeg reda potencija:

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{\sqrt{2n+3}+\sqrt{5n}}.$$

Zadatak 5 Razvijte funkciju $f(x) = \frac{x-2}{x^2-9} + e^x$ u Taylorov red oko $x_0 = 1$ i odredite pripadni interval konvergencije.

$$\begin{aligned}
(c)' &= 0, \quad c \in \mathbb{R} \\
(x^\alpha)' &= \alpha x^{\alpha-1}, \quad \alpha \in \mathbb{R}, x \in \mathbb{R} \\
(\log_a x)' &= \log_a e \cdot 1/x, \quad x > 0 \\
(\ln x)' &= \frac{1}{x}, \quad x > 0 \\
(a^x)' &= a^x \ln a, \quad a > 0, x \in \mathbb{R} \\
(e^x)' &= e^x, \quad x \in \mathbb{R} \\
(\sin x)' &= \cos x, \quad x \in \mathbb{R} \\
(\cos x)' &= -\sin x, \quad x \in \mathbb{R} \\
(\mathrm{tg}\, x)' &= \tfrac{1}{\cos^2 x} \\
(\mathrm{ctg}\, x)' &= -\tfrac{1}{\sin^2 x} \\
(\mathrm{sh}\, x)' &= \mathrm{ch}\, x \\
(\mathrm{ch}\, x)' &= \mathrm{sh}\, x \\
(\mathrm{th}\, x)' &= \tfrac{1}{\mathrm{ch}^2 x} \\
(\mathrm{cth}\, x)' &= -\tfrac{1}{\mathrm{sh}^2 x} \\
(\arcsin x)' &= \tfrac{1}{\sqrt{1-x^2}} \\
(\arccos x)' &= -\tfrac{1}{\sqrt{1-x^2}} \\
(\mathrm{arctg}\, x)' &= \tfrac{1}{1+x^2} \\
(\mathrm{arcctg}\, x)' &= -\tfrac{1}{1+x^2}
\end{aligned}$$

$$\begin{aligned}
\int \mathrm{d}x &= x + c, \quad c \in \mathbb{R}, x \in \mathbb{R} \\
\int x^\alpha \,\mathrm{d}x &= \tfrac{x^{\alpha+1}}{\alpha+1} + c, \quad c \in \mathbb{R}, \alpha \in \mathbb{R} \setminus \{-1\}, x \in \mathbb{R} \\
\int \tfrac{1}{x} \,\mathrm{d}x &= \ln|x| + c, \quad c \in \mathbb{R}, x \in \mathbb{R} \setminus \{0\} \\
\int a^x \,\mathrm{d}x &= \tfrac{a^x}{\ln a} + c, \quad c \in \mathbb{R}, a > 0, x \in \mathbb{R} \\
\int e^x \,\mathrm{d}x &= e^x + c, \quad c \in \mathbb{R}, x \in \mathbb{R} \\
\int \sin x \,\mathrm{d}x &= -\cos x + c, \quad c \in \mathbb{R}, x \in \mathbb{R} \\
\int \cos x \,\mathrm{d}x &= \sin x + c, \quad c \in \mathbb{R}, x \in \mathbb{R} \\
\int \tfrac{\mathrm{d}x}{\cos^2 x} &= \mathrm{tg} x + c, \quad c \in \mathbb{R} \\
\int \tfrac{\mathrm{d}x}{\sin^2 x} &= -\mathrm{ctg} x + c, \quad c \in \mathbb{R} \\
\int \mathrm{sh}\, x \,\mathrm{d}x &= \mathrm{ch}\, x + c \\
\int \mathrm{ch}\, x \,\mathrm{d}x &= \mathrm{sh}\, x + c \\
\int \tfrac{\mathrm{d}x}{\mathrm{ch}^2 x} &= \mathrm{th}\, x + c \\
\int \tfrac{\mathrm{d}x}{\mathrm{sh}^2 x} &= -\mathrm{cth}\, x + c \\
\int \tfrac{\mathrm{d}x}{\sqrt{a^2-x^2}} &= \arcsin \tfrac{x}{a} + c \\
\int \tfrac{\mathrm{d}x}{\sqrt{a^2+x^2}} &= \ln|x + \sqrt{x^2+a^2}| + c \\
\int \tfrac{\mathrm{d}x}{a^2+x^2} &= \tfrac{1}{a} \mathrm{arctg} \tfrac{x}{a} + c \\
\int \tfrac{\mathrm{d}x}{1-x^2} &= \tfrac{1}{2} \ln \tfrac{|x+1|}{|x-1|} + c \\
\int \tfrac{\mathrm{d}x}{x^2-1} &= \tfrac{1}{2} \ln \tfrac{|x-1|}{|x+1|} + c \\
\int \tfrac{\mathrm{d}x}{\sqrt{x^2-a^2}} &= \ln|x + \sqrt{x^2-a^2}| + c
\end{aligned}$$

$$\sin\frac{\pi}{6} = \frac{1}{2},\ \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2},\ \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2},$$
$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2},\ \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2},\ \cos\frac{\pi}{3} = \frac{1}{2}$$