# I. UVOD
## Zadaci za samostalni rad

1. Odredite infimum, supremum, minimum i maksimum skupa

   (a) $A = \{x \in \mathbb{R}\colon\ -3 \leq x < 10\} \cup \langle -7, 5\rangle \cup \{3, 10\}$.
   Rj.: $A = \langle -7, 10]$, $\inf A = -7$, $\min A$ ne postoji, $\sup A = \max A = 10$.

   (b) $B = \left[-\sqrt{2}, \sqrt{2}\right\rangle \cap [-1, 1\rangle \cup \{\sqrt{2}, 8\}$.
   Rj.: $B = [-1, 1\rangle \cup \{\sqrt{2}, 8\}$, $\inf B = \min B = -1$, $\sup B = 8$, $\max B$ ne postoji.

   (c) $C = \{x \in \mathbb{N}\colon x^2 < 4\} \cup \{2, \sqrt{13}\}$.
   Rj.: $C = \langle -2, 2] \cup \{\sqrt{13}\}$, $\inf C = -2$, $\min C$ ne postoji, $\sup C = \max C = \sqrt{13}$.

   (d) $D = \{x \in \mathbb{R}\colon |x| \leq 3\} \cup \{-2, 5\}$.
   Rj.: $D = [-3, 3] \cup \{5\}$, $\inf D = \min D = -3$, $\sup D = \max D = 5$.

   (e) $E = \{x \in \mathbb{R}\colon\ -x^2 < 5x - 14\}$.
   Rj.: $E = \langle -\infty, -7\rangle \cup \langle 2, \infty\rangle$, $\inf E$, $\min E$, $\sup E$, $\max E$ ne postoje.

   (f) $F = \{x \in \mathbb{R}\colon x^2 + x \leq 0\}$.
   Rj.: $F = [-1, 0]$, $\inf F = \min F = -1$, $\sup F = \max F = 0$.

   (g) $G = \{x \in \mathbb{R}\colon\ -6 \leq x \leq 6\} \cup \{8\}$.
   Rj.: $G = [-6, 6] \cup \{8\}$, $\inf G = \min G = -6$, $\sup G = \max G = 8$.

   (h) $H = \{x \in \mathbb{R}\colon x \leq -3\}$.
   Rj.: $H = \langle -\infty, -3]$, $\inf H$, $\min H$ ne postoji, $\sup H = \max H = -3$.

   (i) $I = \{x \in \mathbb{R}\colon x^2 \leq 2\}$.
   Rj.: $I = [-\sqrt{2}, \sqrt{2}]$, $\inf I = \min I = -\sqrt{2}$, $\sup I = \max I = \sqrt{2}$.

2. Pojednostavite izraz

   (a) $\dfrac{|3\sqrt{2} - 2\sqrt{3}| - |2\sqrt{2} - 3\sqrt{3}|}{|\sqrt{2} - 1| - |1 - \sqrt{3}|}$

   (b) $\dfrac{(|x| - 1)(|x| + 2)}{(x + 1)(x + 2)}$

3. Skicirajte grafove sljedećih funkcija

   (a) $f(x) = |2x + 3| - 2$

   (b) $f(x) = ||x| - 5|$

   (c) $f(x) = |x| + |x - 1| + |x - 2|$

4. Riješite sljedeće jednadžbe:

(a) $|2x+3|-4x=0$.
Rj.: $x=\frac{3}{2}$.

(b) $|2x+1|+1-|x-3|=0$.
Rj.: $x_1=\frac{1}{3}, x_2=-3$.

(c) $-7|9-2x|+9=-12$.
Rj.: $x_1=3, x_2=6$.

(d) $|x+1|=3x-9$.
Rj.: $x=5$.

(e) $|x^2+1|=2x$.
Rj.: $x=1$.

(f) $|2x-7|=|4x+6|$.
Rj.: $x_1=\frac{-13}{2}, x_2=\frac{1}{6}$.

5. Riješite sljedeće nejednadžbe:

(a) $5x>3-|4x+2|$.
Rj.: $x\in\langle\frac{1}{9},\infty\rangle$.

(b) $|3x+1|+2x>|4x+5|-1$
Rj.: $x\in\langle\frac{-5}{3},-1\rangle$.

(c) $|3x-1|<2x+5$
Rj.: $x\in\langle\frac{-4}{5},6\rangle$.

(d) $1\leq|-2x+5|\leq 4$
Rj.: $x\in[\frac{1}{2},2]\cup[3,\frac{9}{2}]$.

(e) $|x+2|-|x-2|>1$
Rj.: $x\in\langle\frac{1}{2},\infty\rangle$.

6. Dokažite metodom matematičke indukcije da su sljedeće tvrdnje točne za svaki prirodni broj $n$:

(a) $1+3+5+\cdots+(2n-1)=n^2$

(b) $1^3+2^3+3^3+\cdots+n^3=\left(\frac{n(n+1)}{2}\right)^2$

(c) $1\cdot 2+2\cdot 3+3\cdot 4+\cdots+n(n+1)=\frac{1}{3}n(n+1)(n+2)$

(d) $n^3-n+3$ je djeljivo s 3

(e) $2^{2n}-1$ je djeljivo s 3

(f) $5^n+12n+3$ je djeljivo s 4

7. Pokažite da vrijedi $\binom{n}{k}=\binom{n}{n-k},\ k=1,\ldots,n$

8. Koristeći binomnu formulu odredite

(a) $(x+3a)^4$

(b) $(2y-\frac{1}{2})^5$

9. Odredite koeficijent uz

(a) $x^4$ u izrazu $(2x+3)^6$

(b) $x^8$ u izrazu $(2x^3 - \frac{3}{\sqrt{x}})^5$

(c) $x$ u izrazu $(\frac{1}{\sqrt[3]{x}} - x)^9$

10. Odredite

(a) $|\mathrm{Im}(1+2i) + i\overline{\mathrm{Re}(-4+i)} + 1 + 3i|$

(b) $\dfrac{(1+2i)(2-i)}{(1+i)}$

(c) $\left| i^7 + \dfrac{i^3 - i^{17}}{i^8 - i^{30}} \right|$

11. Odredite trigonometrijski zapis kompleksnog broja

(a) $z = -\sqrt{3} + 3i$

(b) $z = \mathrm{Re}(\overline{\sqrt{3} - i}) + i \cdot \mathrm{Im}(\overline{5 - i})$

12. Odredite

(a) $(1+i)^{33}$

(b) $(-1+i)^{35}(1 - i\sqrt{3})^{152}$

(c) $\dfrac{(\sqrt{3} - i)^{57}(2+2i)^{23}}{(1 - i\sqrt{3})^{13}}$

13. Odredite

(a) $\sqrt[4]{16\,i}$

(b) $\sqrt[3]{-2 - 2i\sqrt{3}}$

(c) $\sqrt{(1 - i\sqrt{3})^7}$

14. Odrediti sva kompleksna rješenja jednadžbe $(3-i)z^3 = -4 + 8i$.