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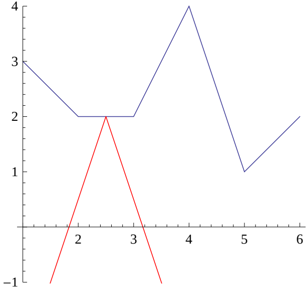

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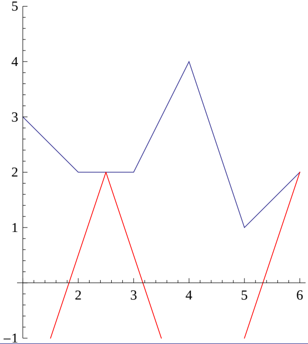

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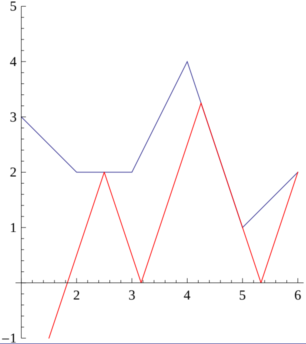

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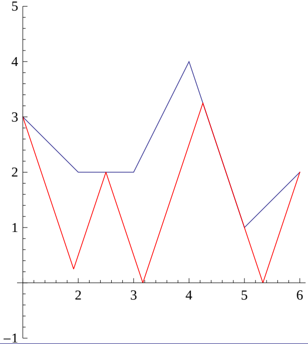

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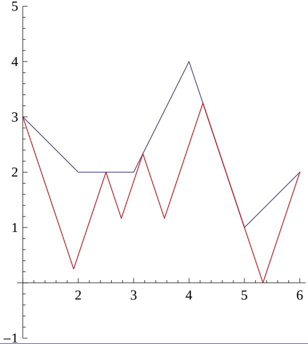

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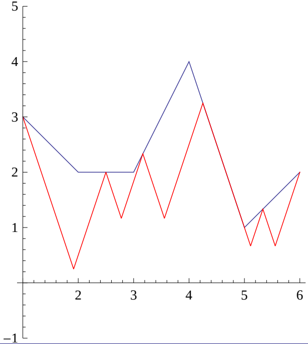

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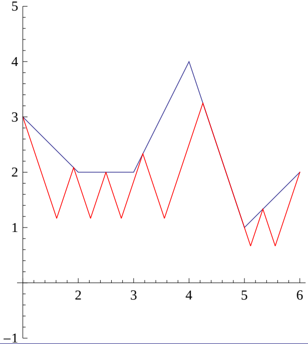

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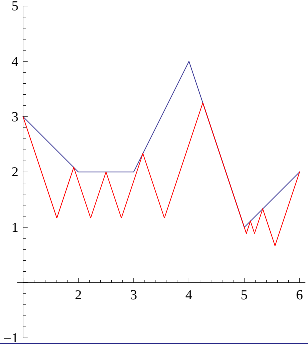

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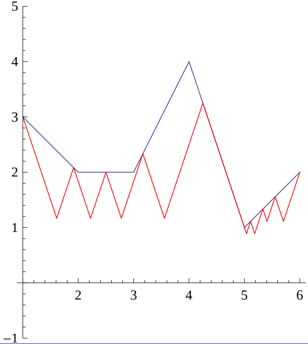

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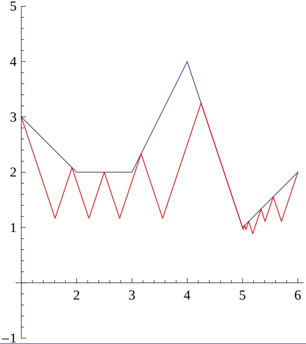

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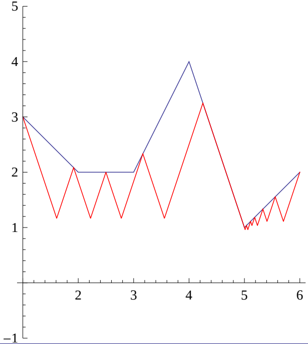

Shubert algoritam (B. Shubert, *A sequential method seeking the global maximum of a function*, SIAM Journal on Numerical Analysis, **9**(1972), 379–388)

- Odrediti pravce $x \mapsto f(a) - L(x-a)$ i $x \mapsto f(b) + L(x-b)$ i točku njihovog sjecišta $(U(a,b,f,L), B(a,b,f,L))$
- Staviti $u_1 = U(a,b,f,L)$. Od dva intervala $[a, u_1]$, $[u_1, b]$ izabrati onaj s manjom B-vrijednosti.
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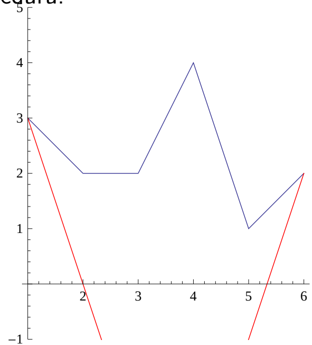

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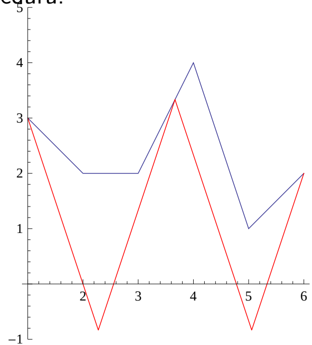

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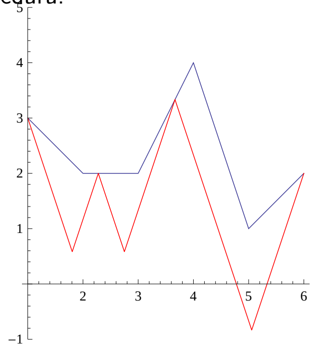

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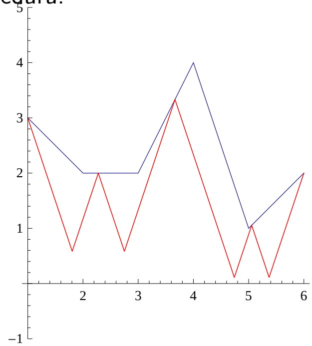

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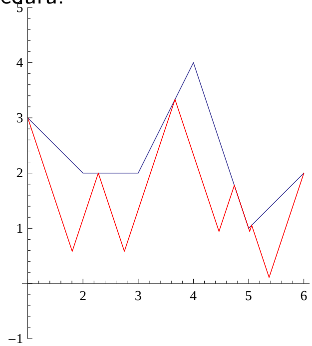

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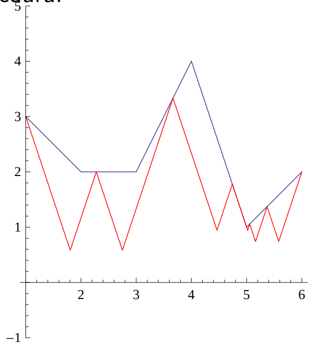

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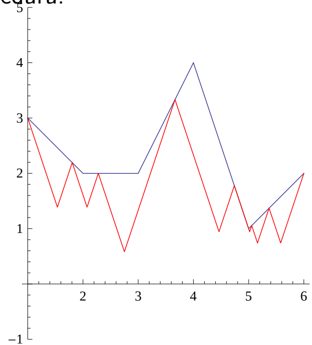

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- Odrediti pravce $x \mapsto f(a) - L(x-a)$ i $x \mapsto f(b) + L(x-b)$ i točku njihovog sjecišta $(U(a,b,f,L), B(a,b,f,L))$
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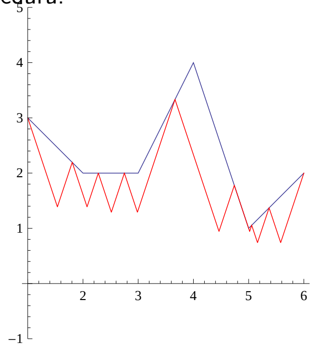

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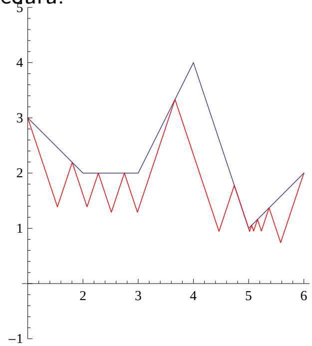

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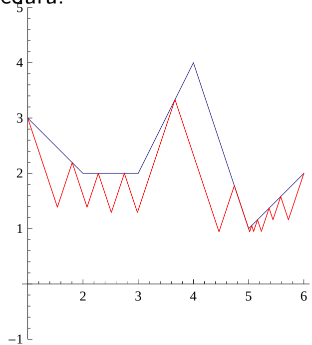

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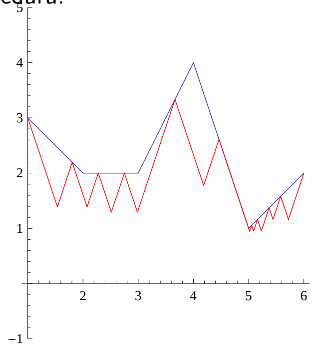

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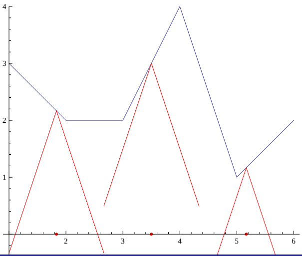

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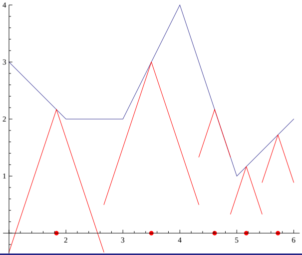

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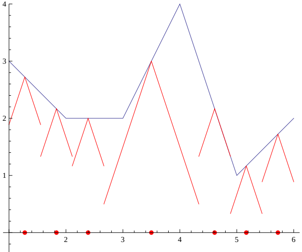

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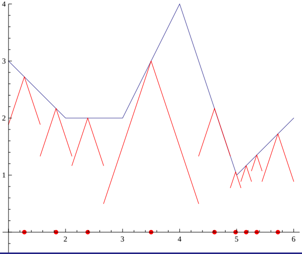

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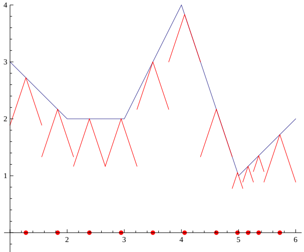

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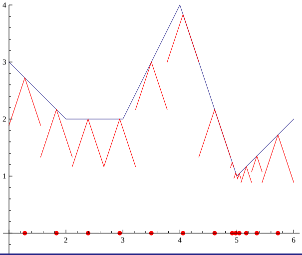

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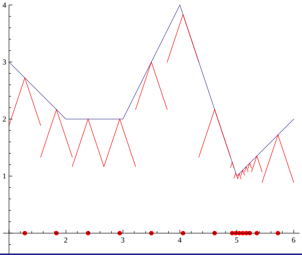

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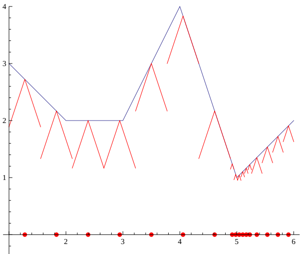

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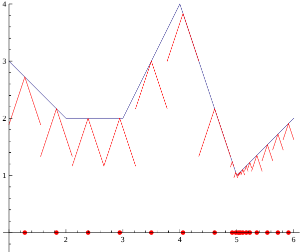

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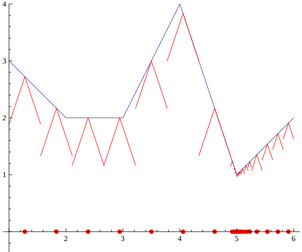

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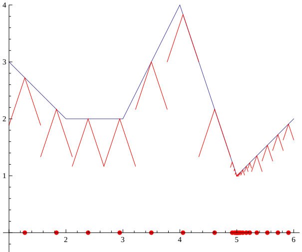
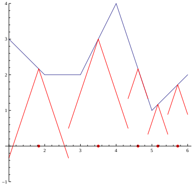
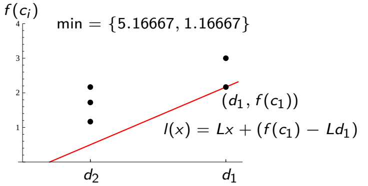


2. iteracija





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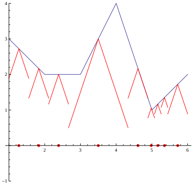
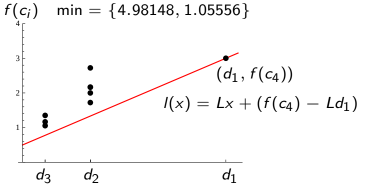
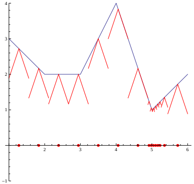
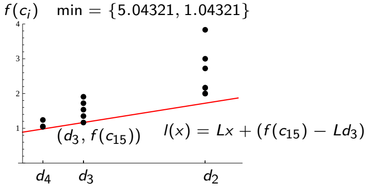


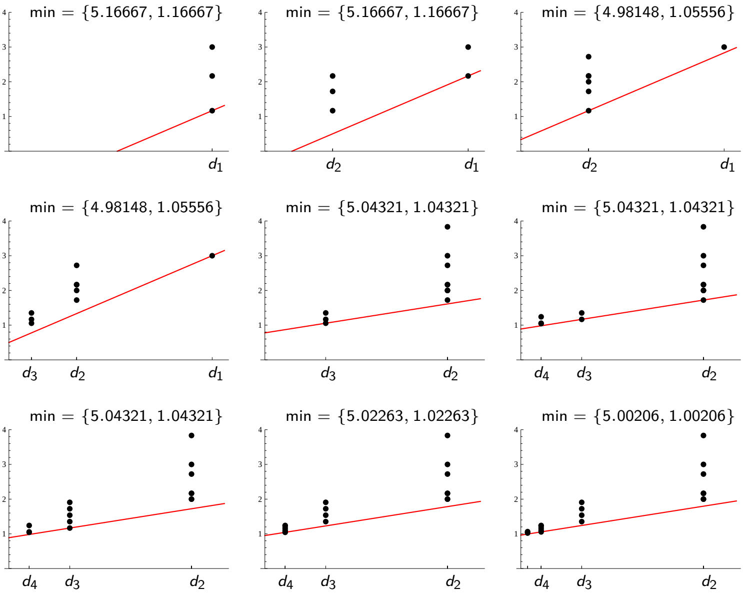


7. iteracija



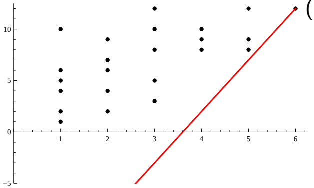


Traženje trenutno optimalnog intervala $\left(d_1 = \frac{b-a}{6},\ d_2 = \frac{b-a}{18},\ d_3 = \frac{b-a}{54},\ d_4 = \frac{b-a}{162}\right)$



Za $\hat{L} = 5$ sve točke leže iznad pravca

$l_1(x) = \hat{L}x + (f(\hat{c}) - \hat{L}\hat{d}) = 5x + (12 - 5 \cdot 6)$



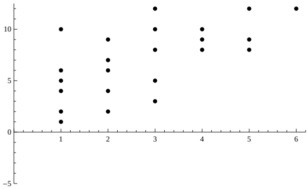
$$f(\hat{c}) - \hat{L}\hat{d} \leq f(c_i) - \hat{L}d_i \quad \text{za sve } i$$

Dakle, točka $\hat{T} = (\hat{d}, f(\hat{c}))$ reprezentira interval na kome se postiže najmanja $\mathcal{B}$-vrijednost.

Traženje trenutno optimalnog intervala ekvivalentno je traženju točke $T_i = (d_i, f(c_i))$, $i = 1, \ldots, n$ kojom prolazi pravac s koeficijentom smjera $L > 0$ i najmanjim odsječkom na ordinati.

Međutim, kao što se može vidjeti ima i boljih pravaca s većim koeficijentom smjera, a koji imaju spomenuto svojstvo:

$$f(c_1) - L_1 d_1 \leq f(c_i) - L_1 d_i \quad \text{za sve } i$$
$$f(c_2) - L_2 d_2 \leq f(c_i) - L_2 d_i \quad \text{za sve } i$$
$$f(c_3) - L_2 d_3 \leq f(c_i) - L_2 d_i \quad \text{za sve } i$$

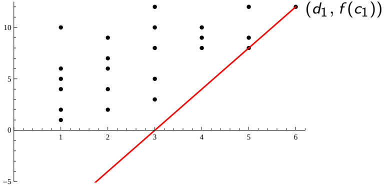


- svaki od navedenih pravaca određuje barem jednu potencijalno optimalnu točku;
- sve dobre točke leže na konveksnoj ljusci;

Međutim, kao što se može vidjeti ima i boljih pravaca s većim koeficijentom smjera,

a koji imaju spomenuto svojstvo:

$$f(c_1) - L_1 d_1 \leq f(c_i) - L_1 d_i \quad \text{za sve } i$$
$$f(c_2) - L_2 d_2 \leq f(c_i) - L_2 d_i \quad \text{za sve } i$$
$$f(c_3) - L_2 d_3 \leq f(c_i) - L_2 d_i \quad \text{za sve } i$$



- svaki od navedenih pravaca određuje barem jednu potencijalno optimalnu točku;
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Međutim, kao što se može vidjeti ima i boljih pravaca s većim koeficijentom smjera, a koji imaju spomenuto svojstvo:

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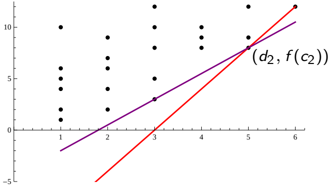
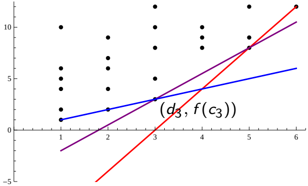
$$f(c_3) - L_2 d_3 \leq f(c_i) - L_2 d_i \quad \text{za sve } i$$



- svaki od navedenih pravaca određuje barem jednu potencijalno optimalnu točku;
- sve dobre točke leže na konveksnoj ljusci;

Međutim, kao što se može vidjeti ima i boljih pravaca s većim koeficijentom smjera, a koji imaju spomenuto svojstvo:

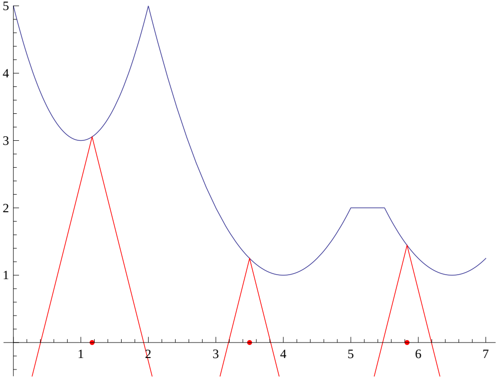
$$f(c_1) - L_1 d_1 \leq f(c_i) - L_1 d_i \quad \text{za sve } i$$
$$f(c_2) - L_2 d_2 \leq f(c_i) - L_2 d_i \quad \text{za sve } i$$
$$f(c_3) - L_2 d_3 \leq f(c_i) - L_2 d_i \quad \text{za sve } i$$



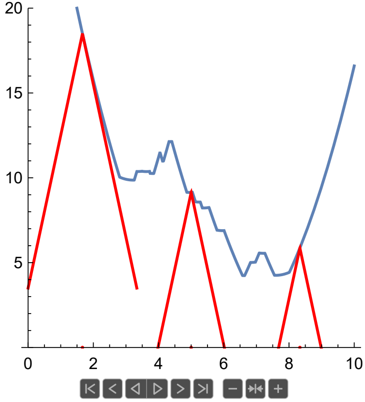
- svaki od navedenih pravaca određuje barem jednu potencijalno optimalnu točku;
- sve dobre točke leže na konveksnoj ljusci;

Zašto nije dovoljno dobro izabrati točke s najmanjom vrijednosti funkcije ?

$$f\colon [1,7] \to \mathbb{R}, \qquad f(x) = \begin{cases} 2(x-1)^2+3, & 0 \le x \le 2, \\ (x-4)^2+1, & 2 \le x \le 5, \\ 2, & 5 \le x \le 5.5, \\ (x-6.5)^2+1, & 5.5 \le x \le 7 \end{cases}$$



- $m = 30$; $a = 0$; $b = 10.$; `y=RandomReal[{a,b}, m];`
- $f : [0, 10] \to [0, 20]$, $f(x) = \underset{i=1,\ldots,m}{\mathrm{med}} (y_i - x)^2$;
- $L = 9$;



Lema (Gablonsky,2001)

Lemma 2

Neka je $f \in Lip_L[a,b]$ *Lipschitz neprekidna funkcija s konstantom* $L > 0$, $\{d_i, c_i : i \in I\}$ *poluširine i centri intervala nastali dijeljenjem intervala* $[a,b]$ *u algoritmu* `DIRECT`, $f_{min} = \min_{i \in I} f(c_i)$ *trenutna vrijednost minimuma.*
Interval $[a_j, b_j]$, $j \in I$ *je potencijalno optimalan onda i samo onda ako je*

$$f(c_j) \le f(c_i), \quad \forall i \in I_0, \tag{1}$$

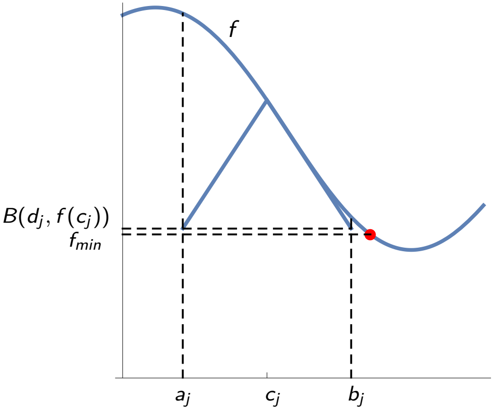
$$\max_{k \in I_L} \frac{f(c_j) - f(c_k)}{d_j - d_k} \le \min_{k \in I_R} \frac{f(c_j) - f(c_k)}{d_j - d_k}, \tag{2}$$

gdje je

- $I_0 = \{i \in I : d_i = d_j\}$ – *indeksi točaka s apscisom* d_j;
- $I_L = \{i \in I : d_i < d_j\}$ – *indeksi točaka koje se na grafikonu nalaze lijevo od točaka s apscisom* d_j;
- $I_R = \{i \in I : d_i > d_j\}$ – *indeksi točaka koje se na grafikonu nalaze desno od točaka s apscisom* d_j.

Ako je $[a_j, b_j]$ potencijalno optimalan intrval sukladno, onda vrijedi

$$\mathcal{B}(d_j, f(c_j)) = f(c_j) - \hat{L}d_j \le f(c_j) \le f_{min}.$$



Slika: Interval koji nije potencijalno optimalan

Lema (Gablonsky,2001)

Lemma 3

Neka je $f \in Lip_L[a,b]$ *Lipschitz neprekidna funkcija s konstantom* $L > 0$, $\epsilon > 0$, $\{d_i, c_i \colon i \in I = \{1, \ldots, n\}$, *poluširine i centri intervala nastali dijeljenjem intervala* $[a, b]$ *u algoritmu* DIRECT *i* $f_{min} = \min\limits_{i \in I} f(c_i)$.
Neka je $S \subset I$ *skup indeksa potencijalno optimalnih intervala i neka je* $\epsilon > 0$.
$\mathcal{B}$*-vrijednost potencijalno optimalnog intervala* $[a_j, b_j]$, $j \in S$ *razlikuje se od aktualnog minimuma* f_{min} *za više od* ϵ *onda i samo onda ako vrijedi*

$$\epsilon \leq \frac{f_{min} - f(c_j)}{|f_{min}|} + \frac{d_j}{|f_{min}|} \min_{k \in I_R} \frac{f(c_j) - f(c_k)}{d_j - d_k}, \qquad \text{ako je } f_{min} \neq 0, \tag{3}$$

odnosno

$$f(c_j) \leq d_j \min_{k \in I_R} \frac{f(c_j) - f(c_k)}{d_j - d_k}, \qquad \text{ako je } f_{min} = 0, \tag{4}$$

gdje je $I_R = \{i \in I \colon d_i > d_j\}$.

Lema (Gablonsky,2001)

Theorem 2

Neka je $f \in Lip_L[a,b]$ *Lipschitz neprekidna funkcija s konstantom* $L > 0$, $\{d_i, c_i \colon i \in I\}$ *poluširine i centri intervala nastali dijeljenjem intervala* $[a,b]$ *u algoritmu* `DIRECT`, $f_{min} = \min_{i \in I} f(c_i)$ *aktualna vrijednost minimuma i* $\epsilon > 0$.
Interval $[a_j, b_j]$, $j \in I$ *je potencijalno optimalan, a njegova* $\mathcal{B}$*-vrijednost razlikuje se od aktualnog minimuma* f_{min} *za više od* ϵ *onda i samo onda ako vrijedi*

$$f(c_j) \le f(c_i), \quad \forall i \in I_0,$$

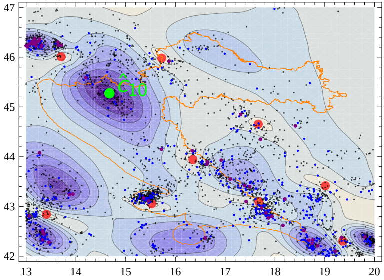
$$\max_{k \in I_L} \frac{f(c_j) - f(c_k)}{d_j - d_k} \le \min_{k \in I_R} \frac{f(c_j) - f(c_k)}{d_j - d_k},$$

$$\begin{cases} \epsilon \le \frac{f_{min} - f(c_j)}{|f_{min}|} + \frac{d_j}{|f_{min}|} \min_{k \in I_R} \frac{f(c_j) - f(c_k)}{d_j - d_k}, & \text{ako je } f_{min} \neq 0, \\ f(c_j) \le d_j \min_{k \in I_R} \frac{f(c_j) - f(c_k)}{d_j - d_k}, & \text{ako je } f_{min} = 0, \end{cases}$$

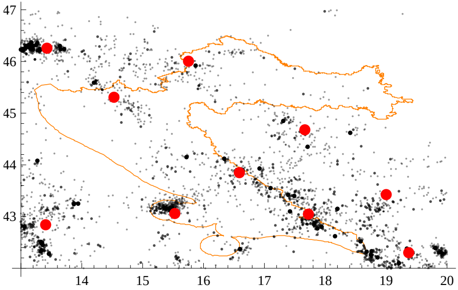
gdje je $I_0 = \{i \in I \colon d_i = d_j\}$, $I_L = \{i \in I \colon d_i < d_j\}$, $I_R = \{i \in I \colon d_i > d_j\}$.

Primjer: detekcija centara seizmičke aktivnosti

(a) `DIRECT` algoritam



(b) k-means algoritam



$$F\colon [13,20]\times[42,47]\to\mathbb{R}_+,$$

$$F(c)=\sum_{i=1}^{m}\min\{d(\hat{c}_1,a_i),\dots,d(\hat{c}_{k-1},a_i),d(c,a_i)\}$$