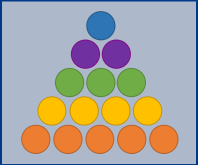

M102 Kombinatorna i diskretna matematika

Vježbe 6

03.04.2018

predavač: Snježana Majstorović

asistent: SM

Binomni koeficijenti

Zadatak 1

Dokažite:

a) $\sum_{k=0}^{n} k\binom{n}{k} = n \cdot 2^{n-1}$

b) $\sum_{k=0}^{n} k^2\binom{n}{k} = n(n+1) \cdot 2^{n-2}$

c) $\sum_{k=0}^{n} \frac{1}{k+1}\binom{n}{k} = \frac{2^{n+1}-1}{n+1}$

d) *(DZ)* $\sum_{k=0}^{n} \frac{k}{2^k}\binom{n}{k} = \frac{n \cdot 3^{n-1}}{2^n}$

e) $\sum_{k=0}^{n} k\binom{n}{k}^2 = n\binom{2n-1}{n-1}$

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Neka je n djeljiv s 8. Tada je broj podskupova čiji je broj elemenata djeljiv s 4 jednak $2^{n-2} + 2^{(n-2)/2}$. Dokažite!

Zadatak 3

Ako je p prost, dokažite $\binom{p}{k} = 0 \mod p, 1 \le k \le p-1$.

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Binomni koeficijenti

Zadatak 4

Dokažite:

a) $\sum_{k=0}^{n} k\,(k-1)\binom{n}{k}\cdot 3^{2-k} = \frac{n(n-1)\cdot 4^{n-2}}{3^{n-2}}$

b) *(DZ)* $\sum_{k=0}^{n} (k+1)\binom{n}{k}^2 = (n+2)\binom{2n-1}{n}$

c) $\sum_{k=0}^{n-1} (n-k)^2\binom{n-1}{n-k} = n\,(n-1)\cdot 2^{n-3}$

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Binomni koeficijenti

(DZ) Zadatak 5

Dokažite:

a) $\sum_{i=0}^{n} (-1)^i \binom{n}{i} \binom{m+n-i}{k-i} = \binom{m}{k}$

b) $\sum_{k=0}^{n} k^3 \binom{n}{k}^2 = n^2 \left[(n-1) \binom{2n-3}{n-2} + \binom{2n-2}{n-1} \right]$

c) $\sum_{k \geq 0} \binom{n}{2k} \binom{2k}{k} \binom{n-2k}{m-k} = \binom{n}{m}^2$

Binomni koeficijenti

Zadatak 6

Dokažite: $\sum_{k=0}^{m} \frac{\binom{m}{k}}{\binom{n}{k}} = \frac{n+1}{n-m+1}$.

Zadatak 7

Neka su $l, s, m, n \in \mathbb{N}, m \geq n$. *Pokažite da* $\forall k$ *za koji suma ima smisla vrijedi:*

$$\sum_k \binom{l}{n+k}\binom{s}{n+k} = \binom{l+s}{l-m+n}$$

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Binomni koeficijenti

Zadatak 8

Dokažite:

$$\sum_k \binom{l}{m+k}\binom{s+k}{n}(-1)^k = (-1)^{l+m}\binom{s-m}{n-l}$$

Multinomni teorem

Multinomni koeficijent:

$$\binom{n}{n_1, n_2, \ldots n_k} = \frac{n!}{n_1!\, n_2! \cdot \ldots \cdot n_k!},$$

pri čemu je $n_1 + n_2 + \ldots + n_k = n$

Specijalno: $\binom{n}{r} = \binom{n}{r, n-r}$

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Multinomni teorem

Teorem 1

Neka je $k, n \in \mathbb{N}$. *tada za svaki* $x_1, x_2, \ldots, x_k \in \mathbb{C}$ *vrjedi:*

$$(x_1 + x_2 + \ldots + x_k)^n = \sum \binom{n}{n_1, n_2, \ldots, n_k} x_1^{n_1} x_2^{n_2} \cdot \ldots \cdot x_k^{n_k}$$

gdje suma ide po svim k*-torkama* $(n_1, n_2, \ldots, n_k), n_i \geq 0, i = 1, \ldots, k$ *takvim da je* $n_1 + n_2 + \ldots + n_k = n$.

Multinomni teorem

Zadatak 9

Odredite koeficijent uz $x_1^3\, x_2\, x_3^2\, x_4^2$ *u razvoju od* $(x_1 + \ldots + x_5)^8$.

Zadatak 10

Odredite koeficijent uz $x_1^4\, x_2\, x_3^7$ *u razvoju od* $(3x_1 - 5x_2 + 2x_3)^{12}$.

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