Model Order Reduction for Linear and Non-Linear Dynamical Systems

D.C. Sorensen

I will discuss two related aspects of model order reduction (MOR). The first involves projection methods derived from Krylov and Rational Krylov theory. These methods are moment matching or interpolatory methods that are closely related to the standard Krylov based iterative methods for linear algebraic systems.

Nonlinear model reduction has seen a number of advances over the past decade. For quite some time, the only reasonable projection method has been Proper Orthogonal Decomposition (POD). This is an orthogonal projection of the dynamical system onto a dominant subspace associated with its trajectories. Although significant dimension reduction can be achieved through this approach, there remains a serious complexity issue associated with evaluation of the projected nonlinear terms. The Discrete Empirical Interpolation Method (DEIM) is one approach to overcoming this deficiency. This approach will be developed and analyzed during this tutorial.

The following is a brief outline of topics that shall be covered.

I. Moment Matching Algorithms for Linear Dynamical Systems

1. Rational approximation and moment matching
   - Model reduction as a rational approximation problem
   - The idea of moment matching
   - Pade approximation - optimality
2. Lanczos and Arnoldi processes
   - Krylov spaces, Controllability and Observability
   - Arnoldi, Symmetric Lanczos, Non-symmetric Lanczos
3. Matching moments for free
   - Fundamental Theorem of Krylov spaces
   - Moment Matching Arnoldi
   - Moment Matching NS-Lanczos
   - Cost and storage comparisons
   - Approximation properties
4. Rational Krylov process
   - Skelton's general interpolation result
   - Rational Krylov
   - Approximation properties

5. Iterative Rational Krylov Methods (IRKA)
   - The Basic IRKA Algorithm
   - Performance
   - Enhancements
6. Error bounds, stability considerations
   - The interpolation problem
   - A-posteriori bounds
   - Optimal interpolation - $\mathcal{H}_2$ bounds

V. Nonlinear Model Reduction

1. Proper Orthogonal Decomposition (POD)
   - Comparison to FEM
   - Examples and properties
2. Discrete Emperical Interpolation(DEIM)
   - Development of DEIM
   - Implementation Issues
   - Error Properties
   - Computational Examples