A Connection between Time Domain Model Order Reduction and Moment Matching

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Abstract

The open literature provides a variety of projection based model order reduction (MOR) methods for linear input-output systems as

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

with a regular matrix $E \in \mathbb{R}^{n\times n}$, as well as matrices $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times p}$ and $C \in \mathbb{R}^{q\times n}$.

Jiang and Chen introduced in [1] a unifying framework for time domain MOR based on general families of orthogonal polynomials. Their method leads to a large linear system of equations

$$Hv = f. \tag{2}$$

The solution vector v is used to compute the projection matrix for dimension reduction. Searching for a reduced order model of order m, (2) has dimension $m \cdot n$. Hence, (2) easily becomes too large to solve even on recent computers.

Since (2) is well structured, and also sparse if E and A are sparse matrices, Jiang and Chen propose an iterative Jacobi-like solution method exploiting this structure. However, it is known that this method often converges slowly. In addition, we can show that the matrix H in (2) is not regular in general.

Exploiting the structure of H we show a connection to a Sylvester equation

$$AVO + EVP = F, \tag{3}$$

where A and E denote the coefficient matrices in (1). O and P are structured, small and depend only on the choice of the family of orthogonal polynomials. The right-hand side f and solution vector v of (2) can be obtained by reshaping F and V stacking their columns on top of each other.

Our approach computes the matrix V by solving a (regularized) Sylvester equation (3) rather than the linear system (2).

Under mild conditions (see, e.g. [2, Section 3.4], [3, Section 2.3]) this Sylvester equation is directly connected to a rational Krylov subspace and hence to interpolatory MOR and moment matching. Depending on the choice of orthogonal polynomials we show that these conditions are always fulfilled.

In that case time domain model order recduction based on orthogonal polynomials is an interpolation method with expansion points only depending on the choice of orthogonal polynomials, but not on characteristics of the system to reduce. Hence, this MOR method is necessarily inferior to other reduction methods choosing their expansion points according to the system (e.g., the iterative rational Krylov algorithm (IRKA)).

References

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[3] T. Wolf, *H_2 Pseudo-Optimal Model Order Reduction*, Dissertation, Technische Universität München, Munich, Germany, 2015.