

Introduction to Riemann-Hilbert problems and applications on zeros of orthogonal polynomials near an algebraic singularity of the measure

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Abstract

In this talk I will make first an introduction to Riemann-Hilbert problems for orthogonal polynomials by presenting the Fokas, Its and Kitaev representation of orthogonal polynomials via matrix Riemann-Hilbert problems. Then I will show these results in particular for classical orthogonal polynomials. The rest of the talk contains an application in the study of the local zero behavior of orthogonal polynomials around an algebraic singularity. It will be shown that the so-called fine zero spacing unravels in the general case, and the asymptotic behavior of neighbouring zeros around the singularity can be described with the zeros of the function linear combination of Bessel functions of the first kind. Moreover, by using Sturm-Liouville theory, I will show the behavior of this linear combination of Bessel functions, thus providing estimates for the zeros in question. The talk is based on a paper (published in *Constructive Approximation* 47 (2018) 407-435) with Tivadar Danka (University of Szeged, Hungary) and an essential part of the study comes from the Riemann-Hilbert problem for the orthogonal polynomial associated to the generalized Jacobi weight which has been considered by Marteen Vanlessen, who used the nonlinear steepest descent method discovered by Deift and Zhou.