Can teaching have an influence on our research work in mathematics?

Nikola Koceić Bilan
Faculty of science, University of Split

Common opinion of university professors is that teaching sometimes disturb research work and takes too much time which can be used in writing papers and studying pure mathematics. In this talk we will discuss how giving classes of general math courses on (under)graduate level can be compatible with our research interest. The author will provide three exciting examples, from his own experience, of how teaching can motivate to start serious researching about problems arisen through the teaching process. Studying of those problems has yielded interesting and unexpected results. The first problem comes from Elementary Mathematics. We consider the curves $y = ax$ and $y = \log_a x$ and their intersecting points for various bases a: Although this problem belongs to the elementary calculus, it turns out that the problem of determining number of these points, for $a \in \langle 0, 1 \rangle$ has been overlooked so far. We prove that this number can be 0; 1; 2 or, even, 3, depending on the base a. The second example is about famous Apollonius' problem. Its generalization along with the solution discover interesting reinterpretation of conics. The third example belongs to Math Analysis. We introduce differentiability of vector and scalar functions of several variables at points which fails to belong to interior of a domain of function. Regardless of the existence of partial derivatives in such points, we consider differentials of a function via matrix notation of linear operator by its directional derivatives along chosen vectors. This approach allows us to apply calculus tool to larger class of functions which have been out of any consideration so far. Consequently, we are able to study math analysis on a different level and discover some unexpected phenomena and spread misconceptions.

References

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