Integralne reprezentacije funkcionalnih redova. Čebyšev–Saigo funkcional

Tibor Poganj

Pomorski fakultet, Sveučilište u Rijeci, Studentska 2, 51000 Rijeka
E-mail: poganj@pfri.hr

Sažetak. Predavanje se sastoji iz tri zasebne cjeline, koje dotiču predavačev istraživački rad u tekućoj kalendarskoj godini:

(i) Integralna reprezentacija Neumannovih redova Besselovih funkcija prve vrste $J_\nu(x)$;
(ii) Integralne reprezentacije nekih vrsta poopćenih Hurwitz–Lerch Zeta funkcija i pripadne nejednakosti;
(u) Čebyšev–Saigoov funkcional nad prostorom sinkronih (asinkronih) parova funkcija.

Ad (i). Promatra se Neumannov red Besselovih funkcija prve vrste oblika

$$\mathfrak{N}_\nu(z) := \sum_{n=1}^{\infty} \alpha_n\, J_{\nu+n}(z), \quad z \in \mathbb{C}\,. \tag{0.1}$$

Pokazuje se da postoji integralna reprezentacija $\mathfrak{N}_\nu(z)$ koja uključuje diferencijalni operator iz Euler–Maclaurinove sumacijske formule, vidi [1]. U domeni valjanosti integralne formule konstruira se klasa funkcija čija je restrikcija na $\mathbb{N}$ niz koeficijenata $(\alpha_n)_{n\in\mathbb{N}}$, [2].

Ad (ii). Definira se proširena Hurwitz–Lerch Zeta funkcija, u smislu Fox–Wrightove jezgre

$$\Phi^{(\rho_1,\cdots,\rho_p,\sigma_1,\cdots,\sigma_q)}_{\lambda_1,\cdots,\lambda_p;\mu_1,\cdots,\mu_q}(z,s,a) = \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{p}(\lambda_j)_{n\rho_j}}{\prod_{j=1}^{q}(\mu_j)_{n\sigma_j}}\, \frac{z^n}{(n+a)^s\, n!}\,; \tag{0.2}$$

integralna reprezentacija i pripadne nejednakosti se diskutiraju iz članaka [3, 4].

Ad (u). Funkcional Čebyševa $T(\cdot,\cdot)$ se promatra na Saigoovim hipergeometrijskim frakcionalnim operatorom integriranja koji smo primijenili na klasi sinkronih (asinkronih) funkcija, nazivajući ga Čebyšev–Saigo funkcional T_S. Formiraju se gornje ograde T, T_S u ovisnosti o glatkosti funkcija u argumentu, [5].

Abstract. The presentation, such that concerns speaker's (recent year) research work, contains three main topics:

- **(i)** Integral representation of Neumann series of first kind Bessel functions $J_\nu(x)$;
- **(ii)** Integral representations of various kind generalized Hurwitz–Lerch Zeta functions and related bounding inequalilites;
- **(u)** Čebyšev–Saigo functional in the class of synchronous (asynchronous) functions.

Ad **(i)**. Consider Neumann series of first kind Bessel functions (0.1). One derives integral expression for $\mathfrak{N}_\nu(z)$ by virtue of the differential operator appearing in Euler–Maclaurinove summation formula, see [1]. In the range of validity of mentioned integral representation we construct of a class of functions which restriction to $\mathbb{N}$ coincides with the coefficients sequence $(\alpha_n)_{n\in\mathbb{N}}$, [2].

Ad **(ii)**. We define unified Hurwitz–Lerch Zeta function, in the sense of Fox–Wright type kernel *via* (0.2). Integral representations and associated bounding inequalities are discussed covering results from [3, 4].

Ad **(u)**. The celebrated Čebyšev functional $T(\cdot,\cdot)$ has been applied to Saigo hypergeometric fractcional integration operator in the class of synchronous (asynchronous) input functions. The resulting operator we call *Čebyšev–Saigo functional* T_S. Certain upper bounds are presented for both T, T_S depending of the smoothnes of argument input functions, [5].

References

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