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Searching for a best LAD-solution of an overdetermined system of linear equations motivated by searching for a best LAD-hyperplane on the basis of given data

We consider the problem of searching for a best LAD-solution of an overdetermined system of linear equations $\mathbf{X}\mathbf{a} = \mathbf{z}$, $\mathbf{X} \in \mathbb{R}^{m \times n}$, $m \geq n$, $\mathbf{a} \in \mathbb{R}^n$, $\mathbf{z} \in \mathbb{R}^m$. This problem is equivalent to the problem of determining a best LAD-hyperplane $\mathbf{x} \mapsto \mathbf{a}^T \mathbf{x}$, $\mathbf{x} \in \mathbb{R}^n$ on the basis of given data (\mathbf{x}_i, z_i) , $\mathbf{x}_i = (x_1^{(i)}, \dots, x_n^{(i)})^T \in \mathbb{R}^n$, $z_i \in \mathbb{R}$, $i = 1, \dots, m$, whereby the minimizing functional is of the form

$$F(\mathbf{a}) = \|\mathbf{z} - \mathbf{X}\mathbf{a}\|_1 = \sum_{i=1}^m |z_i - \mathbf{a}^T \mathbf{x}_i|.$$

An iterative procedure is constructed as a sequence of weighted median problems, which gives the solution in finitely many steps. A criterion of optimality follows from the fact that the minimizing functional F is convex, and therefore the point $\mathbf{a}^* \in \mathbb{R}^n$ is the point of a global minimum of the functional F if and only if $\mathbf{0} \in \partial F(\mathbf{a}^*)$.

Motivation for the construction of the algorithm was found in a geometrically visible algorithm for determining a best LAD-plane $(x, y) \mapsto \alpha x + \beta y$, passing through the origin of the coordinate system, on the basis of the data (x_i, y_i, z_i) , $i = 1, \dots, m$.