$\mathcal{H}_\infty$-Norm Computation for Large-Scale Descriptor Systems

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In this talk we discuss an iterative algorithm for the computation of the $\mathcal{H}_\infty$-norm of continuous-time linear-time invariant descriptor systems

$$E\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t),$$

with transfer function $G(s) := C(sE - A)^{-1}B$. Hereby we focus on the case where E, A, B, and C are large and sparse matrices, so direct methods as described in [1] cannot be applied.

Therefore, we exploit the relationship between the $\mathcal{H}_\infty$-norm and the structured complex stability radius of the transfer function $G(s)$. This is the spectral norm of the smallest complex perturbation Δ that makes the perturbed transfer function

$$G_\Delta(s) := C(sE - (A + B\Delta C))^{-1}B \tag{1}$$

unstable or improper.

Our method is based on *structured ε-pseudospectra* for $G(s)$. This means that we consider *all* perturbations Δ with $\|\Delta\|_2 < \varepsilon$ and analyze how the poles of (1) might move. The set of all these perturbed poles is called the structured ε-pseudospectrum of $G(s)$. To compute the structured complex stability radius we have to find the structured pseudospectrum that touches the imaginary axis. For this calculation we present an algorithm that iterates over the real part of the rightmost pseudopole. This is based on a fast iterative scheme adapted from [2] which relies on the rank-1 representation of certain perturbations. Finally, we illustrate the performance by using real-world examples and also explain pitfalls of the method. The results of this research are available in [3].

[1] Benner, P.; Sima, V.; Voigt, M.: $\mathcal{L}_\infty$-norm computation for continuous-time descriptor systems using structured matrix pencils, *IEEE Trans. Automat. Control*, 57(1):233–238, Jan. 2012.

[2] Guglielmi, N.; Overton, M. L.: Fast algorithms for the approximation of the pseudospectral abscissa and pseudospectral radius of a matrix, *SIAM J. Matrix Anal. Appl.*, 32(4):1166–1192, 2011.

[3] Benner, P.; Voigt, M.: A structured pseudospectral method for $\mathcal{H}_\infty$-norm computation of large-scale descriptor systems, Technical Report MPIMD/12-10, Max Planck Institute Magdeburg, May 2012. Submitted, also available from `http://www.mpi-magdeburg.mpg.de/preprints/`.