

**A problem of Diophantus and Euler on sets in which
 $xy + x + y$ is always a square**

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Diophantus studied the problem of finding numbers such that the product of any two of them increased by the sum of these two gives a square. He found two triples $\{4, 9, 28\}$ and $\{3/10, 7/10, 21/5\}$ satisfying this property. Euler found a quadruple $\{5/2, 9/56, 9/224, 65/224\}$ with the same property and asked if there is an integer solution of this problem. Recently, in joint papers with Clemens Fuchs and Alan Filipin, we proved that there are only finitely many sets of four integers with the above property, and there does not exist such a set consisting of four positive integers. On the other hand, there exist infinitely many rational quintuples with the above property. This result is based on the fact that there are infinitely many rational points on the curve $y^2 = -(x^2 - x - 3)(x^2 + 2x - 12)$.