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Fractional Pearson Diffusion

Abstract

Pearson diffusions have stationary distributions of Pearson type. They includes Ornstein-Uhlenbeck, Cox-Ingersoll-Ross, and several others well-kown processes. Their stationary distributions solve the Pearson equation, developed by Pearson in 1914 to unify some important classes of distributions (e.g., normal, gamma, beta). Their eigenfunction expansions involve the traditional classes of orthogonal polynomials (e.g., Hermite, Laguerre, Jacobi). We develop fractional Pearson diffisions, constracting by a non-Markovian inverse stable time change. Their transition densities are shown to solve a time-fractional analogue to the diffusion equation with polinomial coefficients. Because this process is not Markovian, the stochastic solution provides additional information about the movement of particles that diffuse under this model.

Anomalous diffusions have proven useful in applications to physics, geophysics, chemistry, and finance