Variational Principles and their Applications for Hyperbolic Quadratic Eigenvalue Problems

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Abstract

The hyperbolic quadratic eigenvalue problem (HQEP) was shown to admit Courant-Fischer type min-max principles in 1955 by Duffin and Cauchy type interlacing inequalities in 2010 by Veselić. It can be regarded as the closest analogue (among all kinds of quadratic eigenvalue problems) to the standard Hermitian eigenvalue problem (among all kinds of standard eigenvalue problems).

In this talk, we will show several kinds of min-max principles which are the existing or our new ones. As their applications, we obtain several results of perturbation analysis on the eigenvalues. Also, we justify the natural generalization of the Rayleigh-Ritz procedure with these optimization principles, and then we extend various current optimization approaches — steepest descent/ascent and nonlinear conjugate gradient type methods for the Hermitian eigenvalue problem — to calculate a few extreme eigenvalues. A convergence analysis is given for the steepest descent/ascent methods. The analysis reveals the intrinsic quantities that control convergence rates which implies choices for effective preconditioners. Numerical examples are presented to demonstrate the proposed theory and algorithms.

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