Welsh Institute of Mathematical and Computational Sciences (WIMCS) and LANCS are organizing Workshop

MODERN TRENDS IN MARKOV PROCESSES AND QUEUEING THEORY

15 February 2013

Programme and abstracts

All sessions to be held in Room M/1.10, Mathematical Institute, Senghennydd Road, Cardiff

8.30 - 8.55 - **Coffee and registration in Room M/1.10**

9.00 - 9.40 - **Dr. Vince Knight, Cardiff University, UK**
(30 minutes lecture, plus 10 minutes for questions/discussion)

On an approximate solution to an equation that appears in queueing games

Abstract

At the intersection of game theory and queueing theory lie queueing games. One approach to modelling queueing games is to formulate them as routing games. When doing so arrival rates at queueing systems correspond to strategies of non-atomic players. To obtain these strategies, equations need to be solved that cannot be done so exactly. In this talk I will present some work on an observation as to the form of these solutions. In particular, I will show that when considering $M/M/c$ queues an approximate solution can be found that is linear in the number of servers. The talk will discuss the usefulness of such an approximation, as well as prove an upper bound on its precision.

9.45 - 10.25 - **Professor Alexander Veretennikov, University of Leeds, UK**
(30 minutes lecture, plus 10 minutes for questions/discussion)

On convergence rates in Erlang-Sevastyanov type problem

Abstract

Polynomial convergence rates in total variation are established in Erlang-Sevastyanov's type problem with an infinite number of servers under appropriate assumptions on the intensity of serving.

10.30 - 10.55 - **Professor Vladimir Anisimov, Innovation, Quintiles and School of Mathematics and Statistics, University of Glasgow, UK**
(30 minutes lecture, plus 10 minutes for questions/discussion)

Enrolment in multicentre trials: new trends and perspectives for queuing models

Abstract

Patient enrollment in multicentre clinical trials is described by multidimensional Poisson processes which is similar to modeling customer's flows in multichannel queueing models. However, to mimic the behaviour of real trials and account for multiple centre's effect, it is necessary to introduce several levels of stochastic hierarchy: random variation in enrolment rates of Poisson processes described by a gamma distribution (Poisson-gamma enrollment model [1]) and random delays of enrolment start-up in each center using some type of distributions (uniform, gamma, beta). The next level of hierarchy is modelling the process of various events associated with already enrolled patients, e.g., if treatment takes a long time, the number of followed-up patients in centres or the process of different events (recurrence, death and lost-to-follow-up in oncology trials). For this purpose it is proposed to use "evolving" stochastic processes [2]. In particular, for modelling various events associated with followed-up patients the absorbing finite Markov processes that start at times of patient arrivals are used. In the talk some of these hierarchic models are investigated, in particular, the predictive distributions, stationary characteristics and asymptotic approximations for the number of followed-up patients and the number of different events associated with these patients are derived using analytic technique. Some implementations in real trials are discussed.
These directions open wide opportunities for use in modern multichannel queueing models for modelling flows of various events associated with different type's customers.

References

1. Anisimov, V., Fedorov, V. (2007). Modeling, prediction and adaptive adjustment of recruitment in multicentre trials. *Statistics in Medicine*, **26, 27**: 4958–4975.
2. Anisimov, V. (1991). Limit theorems for evolving accumulation processes, *Theor. Probability and Math. Statist.*, **43**: 5-11.

11.45 - 12.25 - **Professor Jeff Griffiths, Cardiff University, UK**
(30 minutes lecture, plus 10 minutes for questions/discussion)

Approximations for Time-dependent Queues

Abstract

There have been large numbers of publications over many years attempting to find analytic, numerical, or approximate solutions to queueing systems where the parameters are time dependent. Many of the solutions produced only work well for specific ranges of the parameters, such as at low levels of traffic intensity. In situations with high levels of

congestion, it frequently occurs that demand levels exceed capacity for periods of time. In such cases, steady state theory breaks down, resulting in predictions of infinite queue lengths and waiting times. In reality, of course, queue lengths (although sometimes very large) subside when demand levels again fall below capacity. It is important for planning purposes to be able to predict the "time lag" between the peak of demand and the peak of the queue length (or waiting time). Discussions with OR practitioners (who often do not possess high levels of mathematical skills) have indicated the need for simple approximate solutions to time-dependent queueing situations. This is the task I set myself.

12.30 - 13.25 - **Lunch**

13.30 - 14.10 - **Dr. Julie Vile, Cardiff University, UK**
(30 minutes lecture, plus 10 minutes for questions/discussion)

Managing time-varying demand when setting service requirements in priority service systems

Abstract

Despite the fact that demand for emergency assistance is heavily time-dependent, ambulance services are expected to provide a consistently high quality of service with timely responses to all emergency incidents, raising the question of how staffing decisions should be adapted in view of controlling customer's waiting times. Previous studies have proposed approximate and numerical methods to staff time-dependent service systems, and this presentation explains how these methodologies can be extended to priority service systems with two customer classes presenting time-varying demands.
When performing transient analysis of system behavior, previous works have incorporated preemptive or exhaustive service disciplines at shift boundaries; yet this presentation highlights that it is inappropriate to apply these disciplines at artificially imposed shift boundaries (i.e. those created for the purpose of obtaining minimum staffing requirements for short periods, that later form the basis of a shift schedule where staff are employed to work shifts spanning several short periods in succession). We accordingly define instantaneous transitions that map the change in the composition of customers present resulting from various workforce adjustments at novel 'dummy' shift boundaries. These dummy boundaries allow excess staff to leave the system, new staff to join the workforce and a set of base staff to simultaneously serve customers without interruption. Application of the methodology with Welsh Ambulance Service Trust (WAST) data illustrates how the methodology may be used to recommend staffing levels that satisfy government performance targets.

14.15 - 14.55 - **Dr. Denis Denisov, University of Manchester, UK**
(30 minutes lecture, plus 10 minutes for questions/discussion)

Corrected heavy traffic approximation for the single-server queue

Abstract

We consider steady state waiting time for the single server queue. Heavy traffic approximation states that when traffic intensity *rho* close to 1 the distribution of the waiting time (at appropriate scale) is close to exponential. We provide correction terms to this approximation assuming that only finite number of moments of service and interarrival times exist. This extends well known results of Borovkov (1964) and Siegmund (1979)

15.00 - 15.25 - **Tea/biscuits**

15.30 - 16.10 - **Profesor Alexey Kulik, Mathematical Institute of Academy of Sciences of Ukraine, Ukraine**
(30 minutes lecture, plus 10 minutes for questions/discussion)

Ergodicity, mixing bounds, and statistical inference for Fisher-Snedecor diffusion

Abstract

The talk is based on the joint paper with N.N.Leonenko (1).
We consider the Fisher-Snedecor diffusion; that is, the Kolmogorov-Pearson diffusion with the Fisher-Snedecor invariant distribution. In the non-stationary setting, we give explicit quantative rates for *the convergence rate* of respective finite-dimensional distributions to that of the stationary Fisher-Snedecor diffusion, and for *the β-mixing coefficient* of this diffusion. Our way to treat this problem is based on the general theory developed for (possibly non-symmetric and non-stationary) Markov processes, although there is a substantial novelty in the form taken by the *Lyapunov-type condition* (typical in the field) in our setting.
As an application, we prove the law of large numbers and the central limit theorem for additive functionals of the Fisher-Snedecor diffusion. The modified version of the Lyapunov-type condition, mentioned above, implies a substantial difference between the asymptotic properties of the finite-dimensional distributions themselves and their continuous-time averages. In particular, in the continuous-time version of our CLT, the observable functional *may fail to be square integrable* w.r.t. the invariant distribution of the process. This interesting effect seemingly has not been observed in the literature before.
Finally, we apply the above results and provide a statistical analysis for the Fisher-Snedecor diffusion. We prove that respective empirical moments and empirical covariances are P-consistent, asymptotically normal, and asymptotically unbiased. Then, using the method of moments, we construct P-consistent and asymptotically normal estimators for the parameters given either the discrete-time or the continuous-time observations of a non-stationary version of the Fisher-Snedecor diffusion.

In the non-stationary setting we give explicit quantitative rates for the convergence rate of respective finite-dimensional distributions to that of the stationary Fisher-Snedecor diffusion, and for the β-mixing coefficient of this diffusion. As an application, we prove the law of large numbers and the central limit theorem for additive functionals of the Fisher-Snedecor diffusion and construct P-consistent and asymptotically normal estimators for the parameters of this diffusion given its non-stationary observation.

1. Kulik, A.M., Leonenko, N.N.. Ergodicity and mixing bounds for the Fisher-Snedecor diffusion. *To appear in Bernoulli.*

16.15 - 16.55 - **Dr. Nenad Šuvak, University of Osijek, Croatia**
(30 minutes lecture, plus 10 minutes for questions/discussion)

On spectral properties and statistical analysis of heavy-tailed Pearson diffusions

Abstract

The talk is based on the joint papers with Prof. N.N. Leonenko and Prof. F. Avram.
We consider the problem of spectral representation of transition density for a class of diffusion processes with heavy-tailed Pearson type invariant distributions of Reciprocal gamma, Fisher-Snedecor and Student type. As opposed to the classical Pearson diffusions (Ornstein-Uhlebeck, Cox-Ingersoll-Ross and Jacobi diffusions), these heavy-tailed Pearson diffusions have a continuous part of the spectrum which has a significant impact on statistical problems related to these processes. We present the spectral representation of the transition density of these diffusions which involves a finite number of discrete eigenfunctions (classical orthogonal polynomials) as well as a continuous part.
Furthermore, we consider the problem of parameter estimation for these diffusions. We propose moments based estimators and prove their consistency and asymptotic normality. By using the spectral representation of the transition density we calculate the explicit expressions for covarinaces in the asymptotic normality framework.

1. Leonenko, N. N., Šuvak, N. (2010). Statistical inference for reciprocal gamma diffusion process. *J. Statist. Plann. Inference*, **140**: 30–51.

2. Leonenko, N. N., Šuvak, N. (2010). Statistical inference for Student diffusion process. *Stoch. Anal. Appl.*, **28**: 972–1002.

3. Avram, F., Leonenko, N. N., Šuvak, N. (2011). Spectral representation of transition density of Fisher-Snedecor diffusion. *submitted, extended version is published on arXiv - arXiv:1007.4909v1.*

17:00 - **Closing**

14 February, 19.30 - **Dinner (Bellinis restaurant, 1 Park Place, City Centre, Cardiff CF10 3DP)**

J. Griffiths and N. Leonenko, Organisers